Matching Problems

A 3/2-approximation algorithm for the student-project allocation problem with ties

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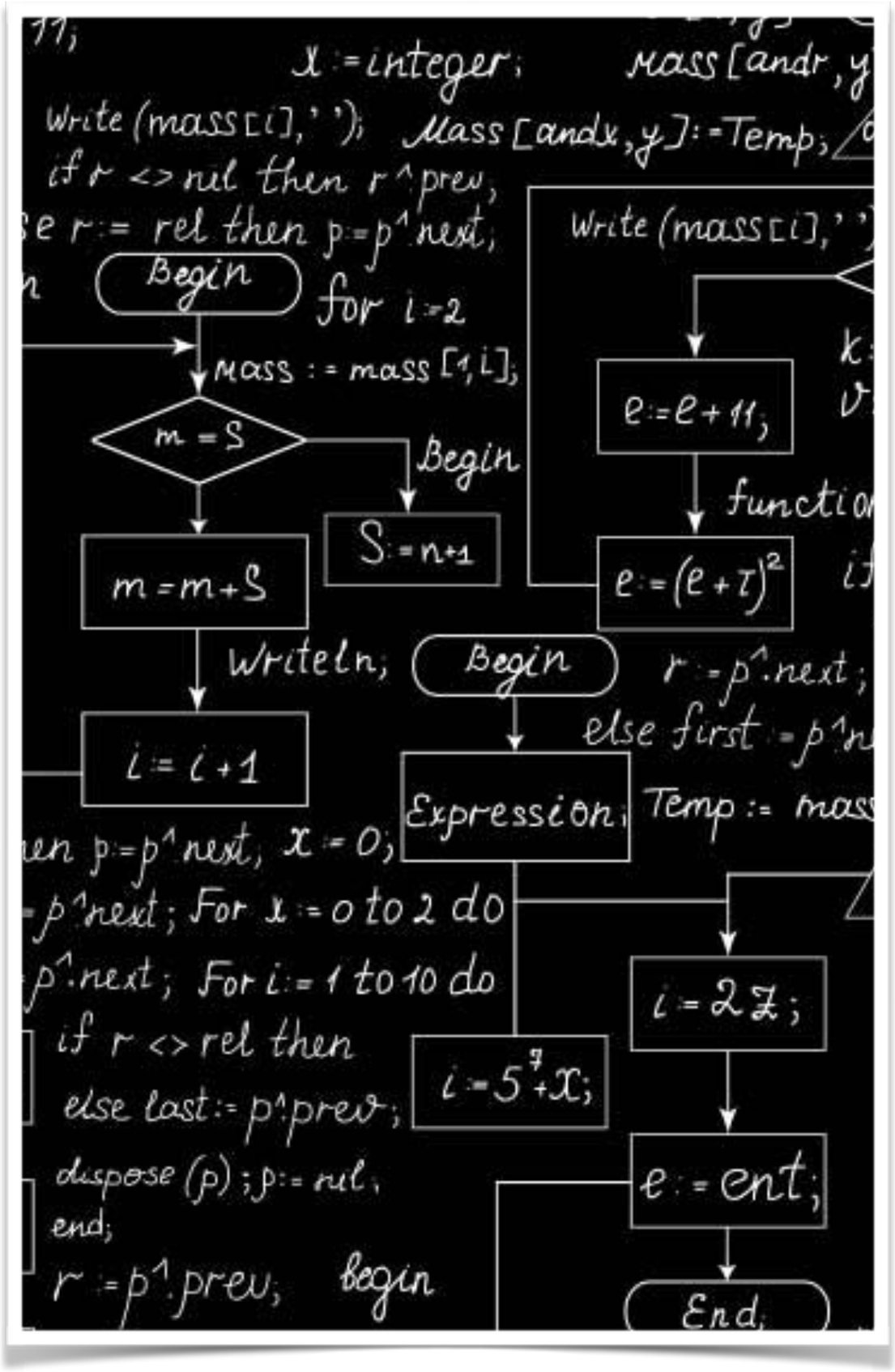
Outline

- Algorithms
- Matching problems
- Stable matchings
- Finding maximum stable matchings

Algorithms

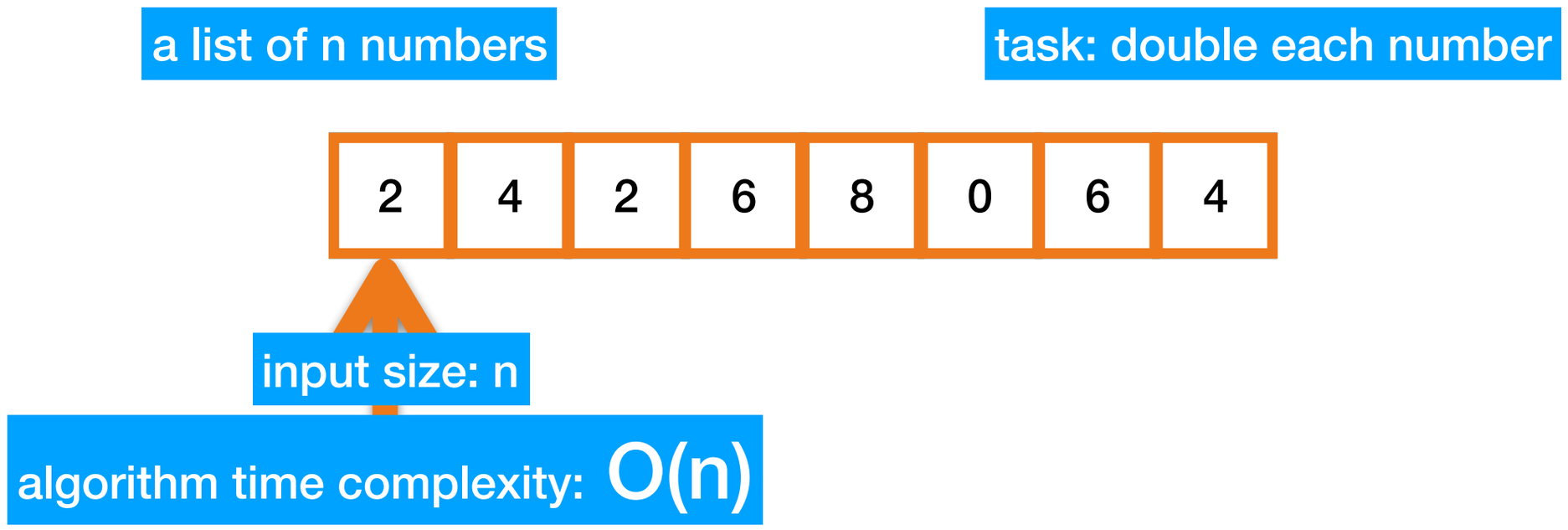

x := integer;
mass [andr, y
Write (mass [i], ' ');
Mass [andx, y] := Temp;
if r <> nil then r^.prev,
e r := rel then p:=p^.next;
Begin
for i:=2
mass := mass [1, i];
m = S
Begin
S := n+1
m = m+S
Writeln;
Begin
i = i+1
Expression;
Temp := mass
en p:=p^.next, x = 0;
=p^.next; For x := 0 to 2 do
p^.next; For i := 1 to 10 do
if r <> rel then
else last := p^.prev;
dispose (p); p := nil,
end;
r := p^.prev, begin
Write (mass [i], ' ')
e := e+11;
e := (e+1)^2
r := p^.next;
else first := p^n
i = 27;
i := 5^7+x;
e := ent;
End.

Algorithms

- What is an algorithm? - a list of instructions given to a computer in order to solve a problem

- The **time complexity** of an algorithm is a measure of how the number of operations grows with respect to input size

- Example:

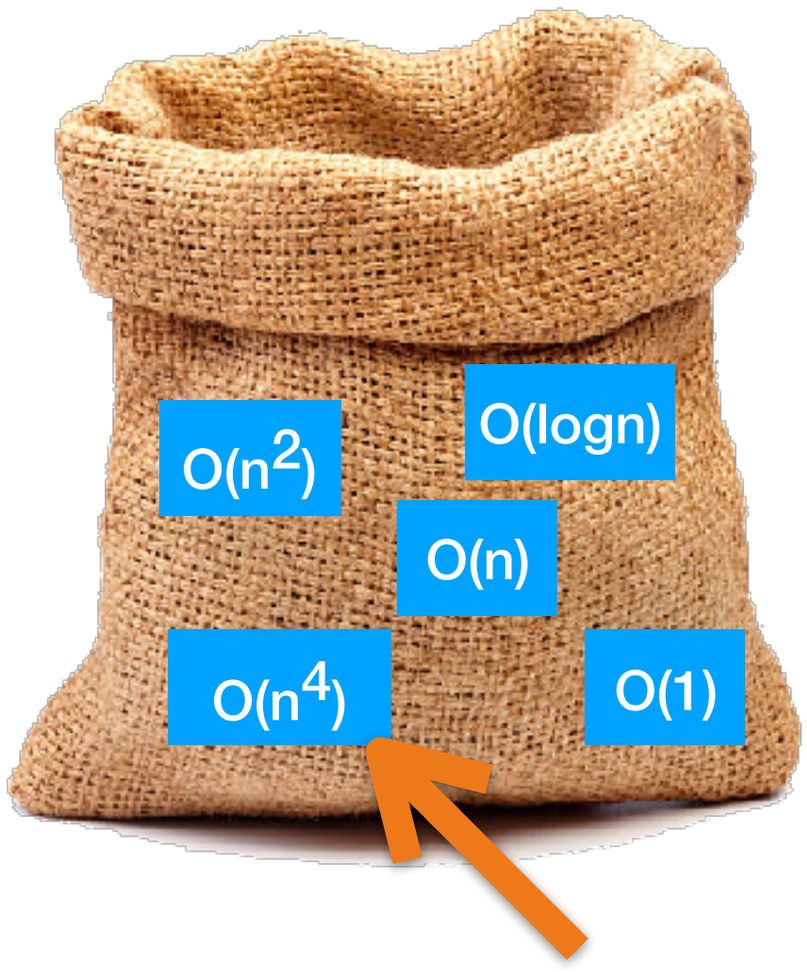


Efficient algorithms

efficient

polynomial time

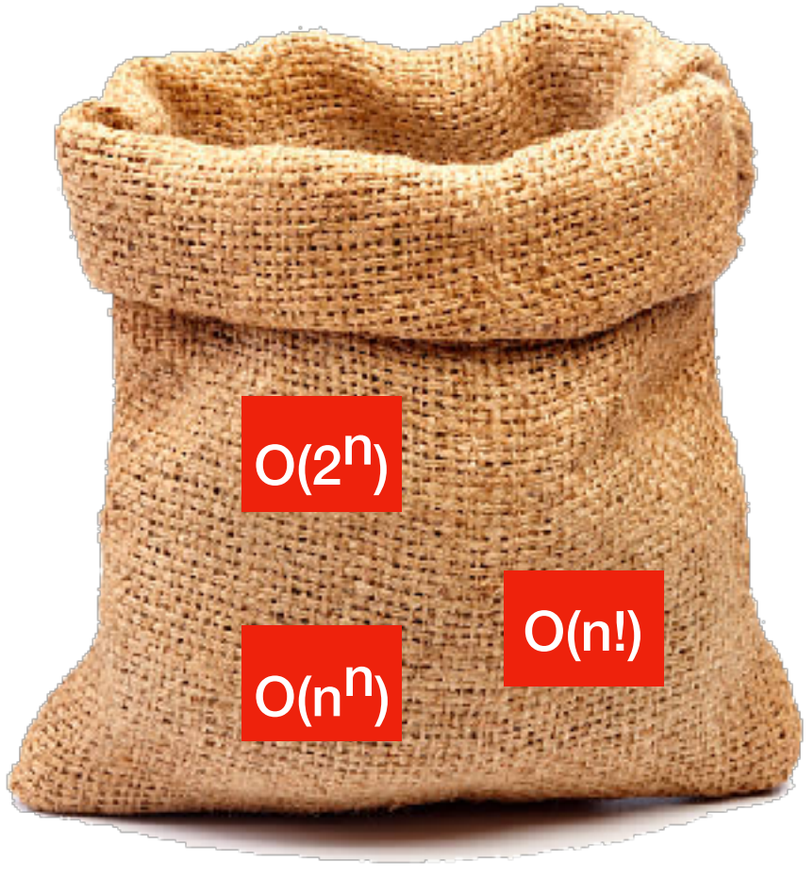
fast in general



inefficient

exponential time

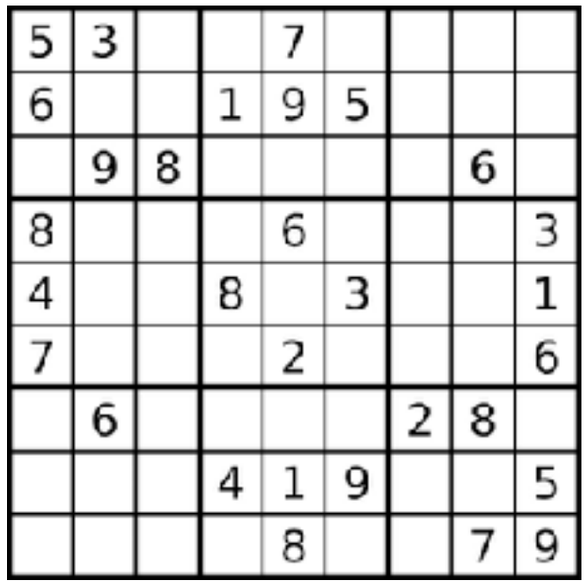
slow in general



NP hardness

- If a problem is **NP hard**, then there is no known efficient algorithm that can solve it

- Is it possible for an efficient algorithm to solve an **NP hard** problem?



This is one of the biggest open questions in computing science

http://www.claymath.org/millennium-problems

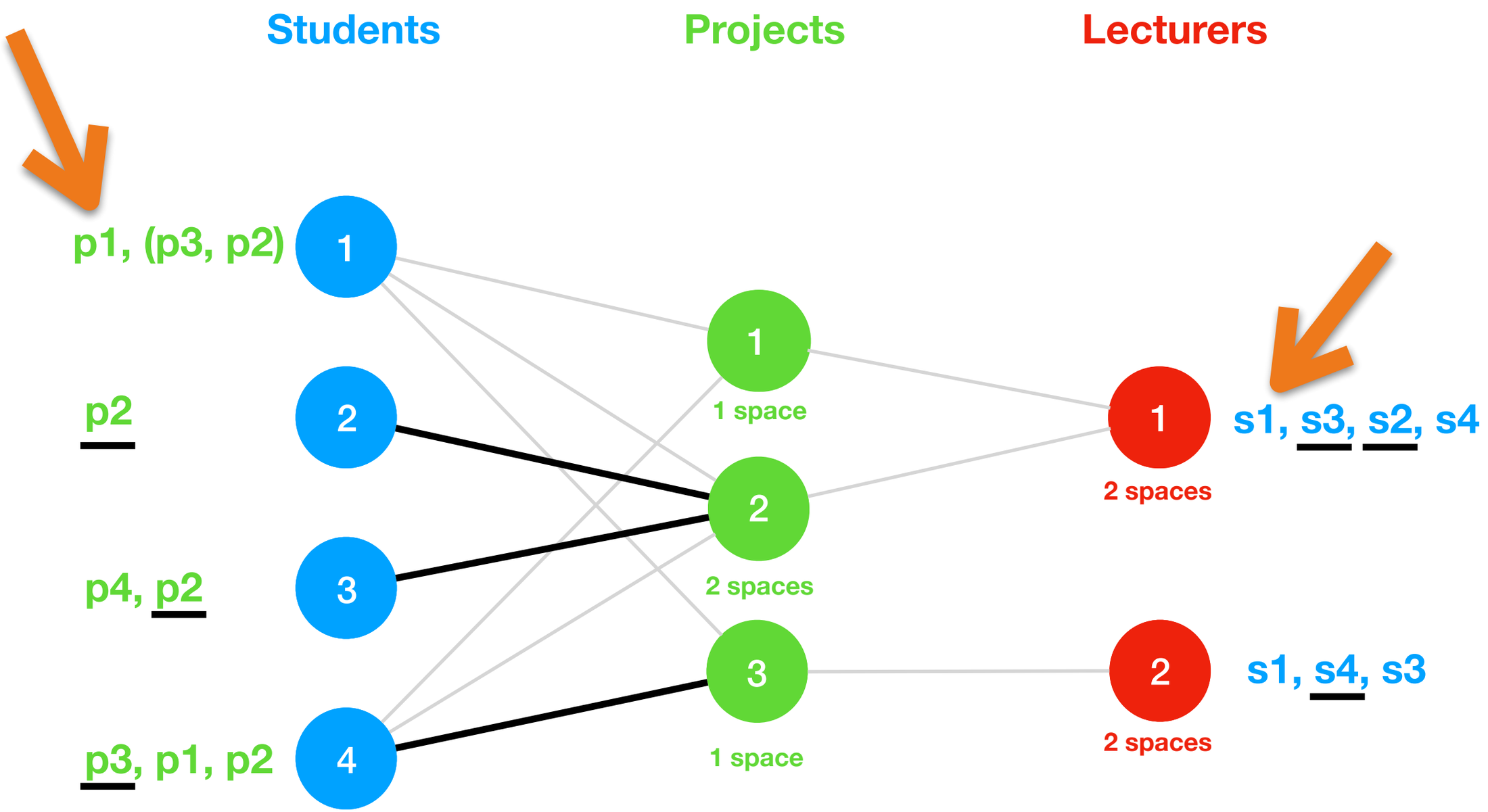
$1,000,000 prize

Matching Problems

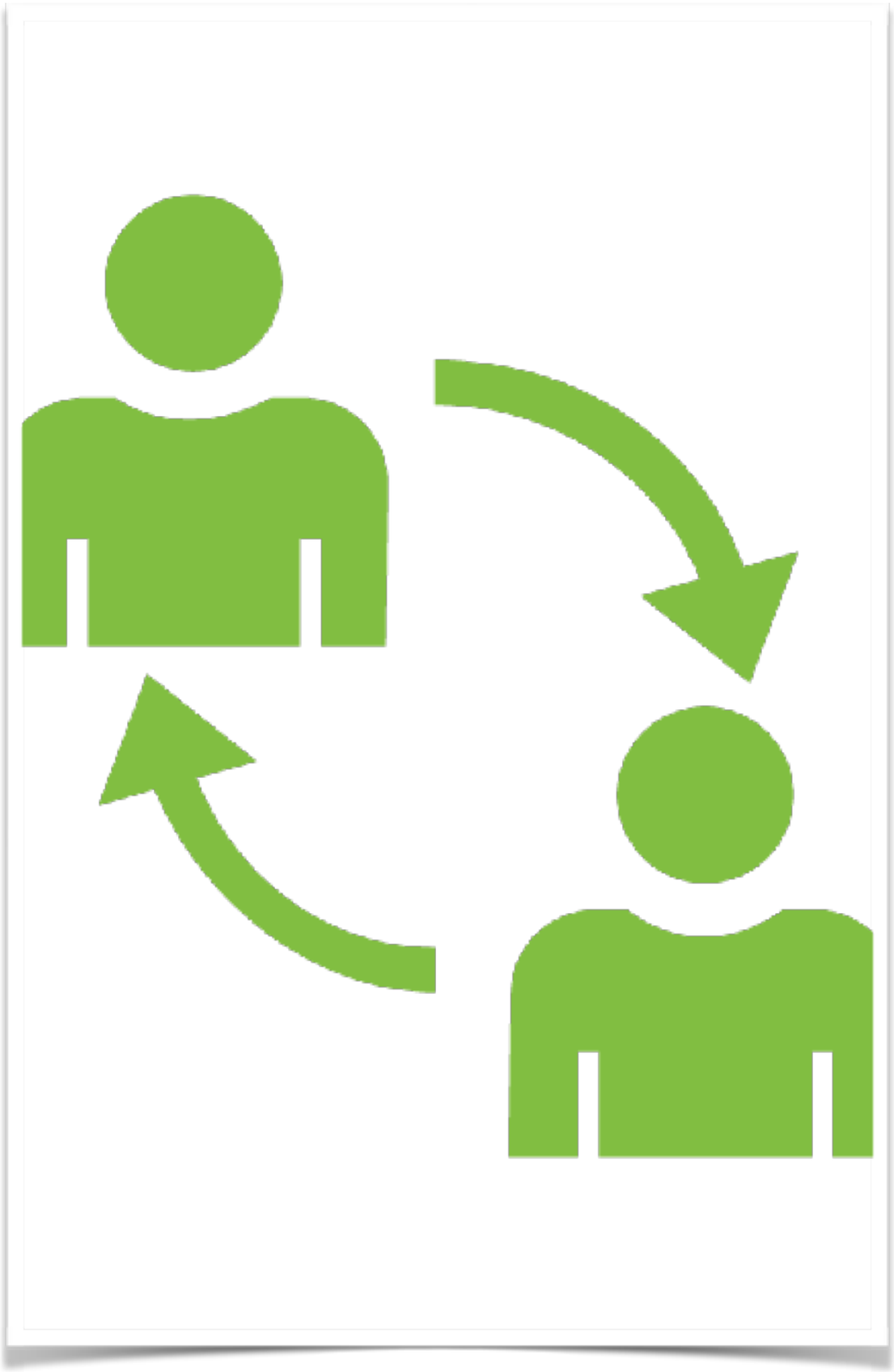
Matching problems

- Assign one group of things to another group of things
- Based on preferences

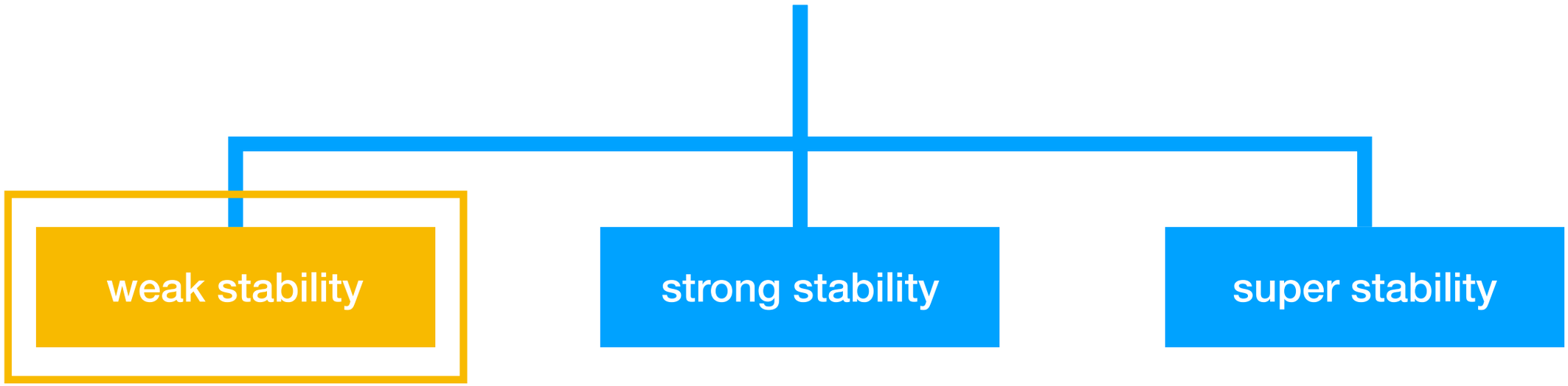
Student-project allocation problem



Stable matching



Stable matching



No Blocking pairs: both agents benefit

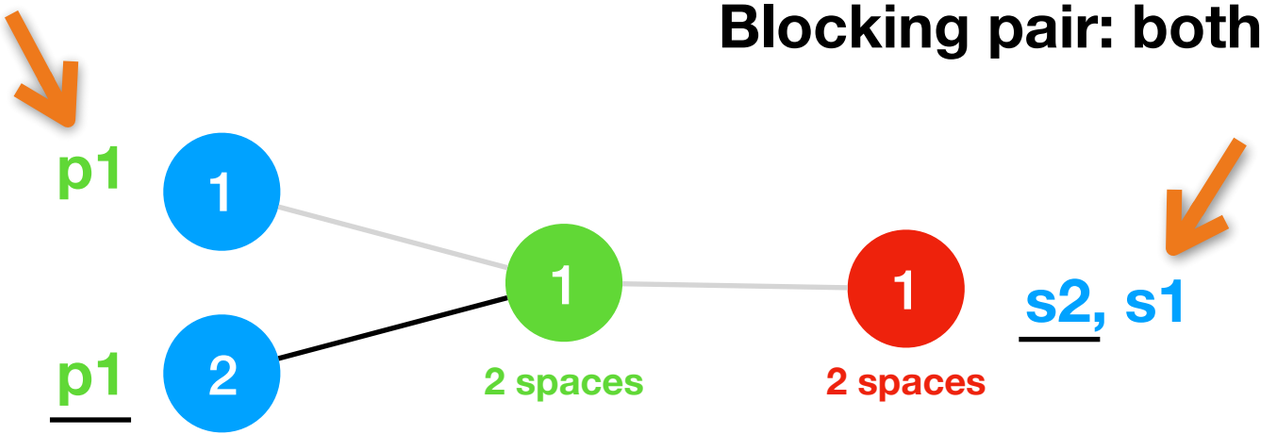
No Blocking Pairs: one agent benefits, the other is indifferent

No Blocking Pairs: neither agent is disadvantaged

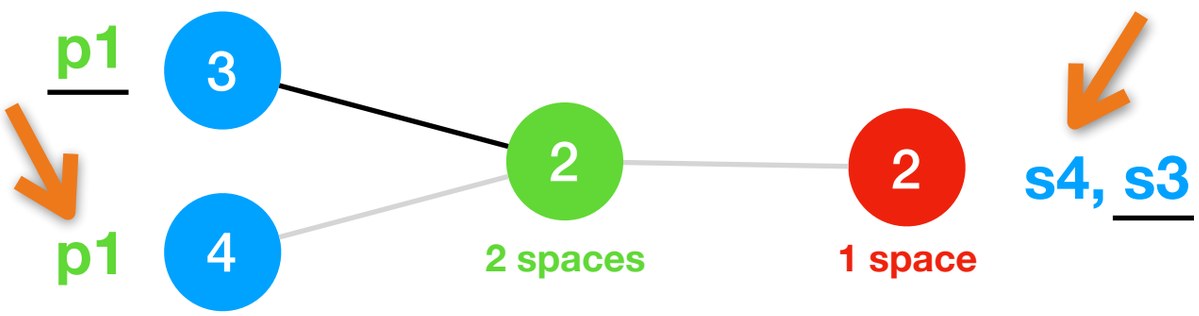
- A **stable matching** is a matching with no blocking pairs

weak stability

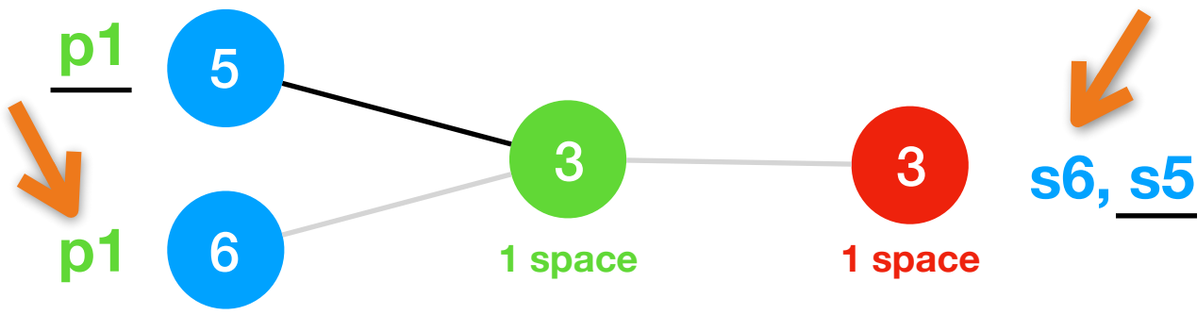
Blocking pair: both agents benefit



project and lecturer undersubscribed

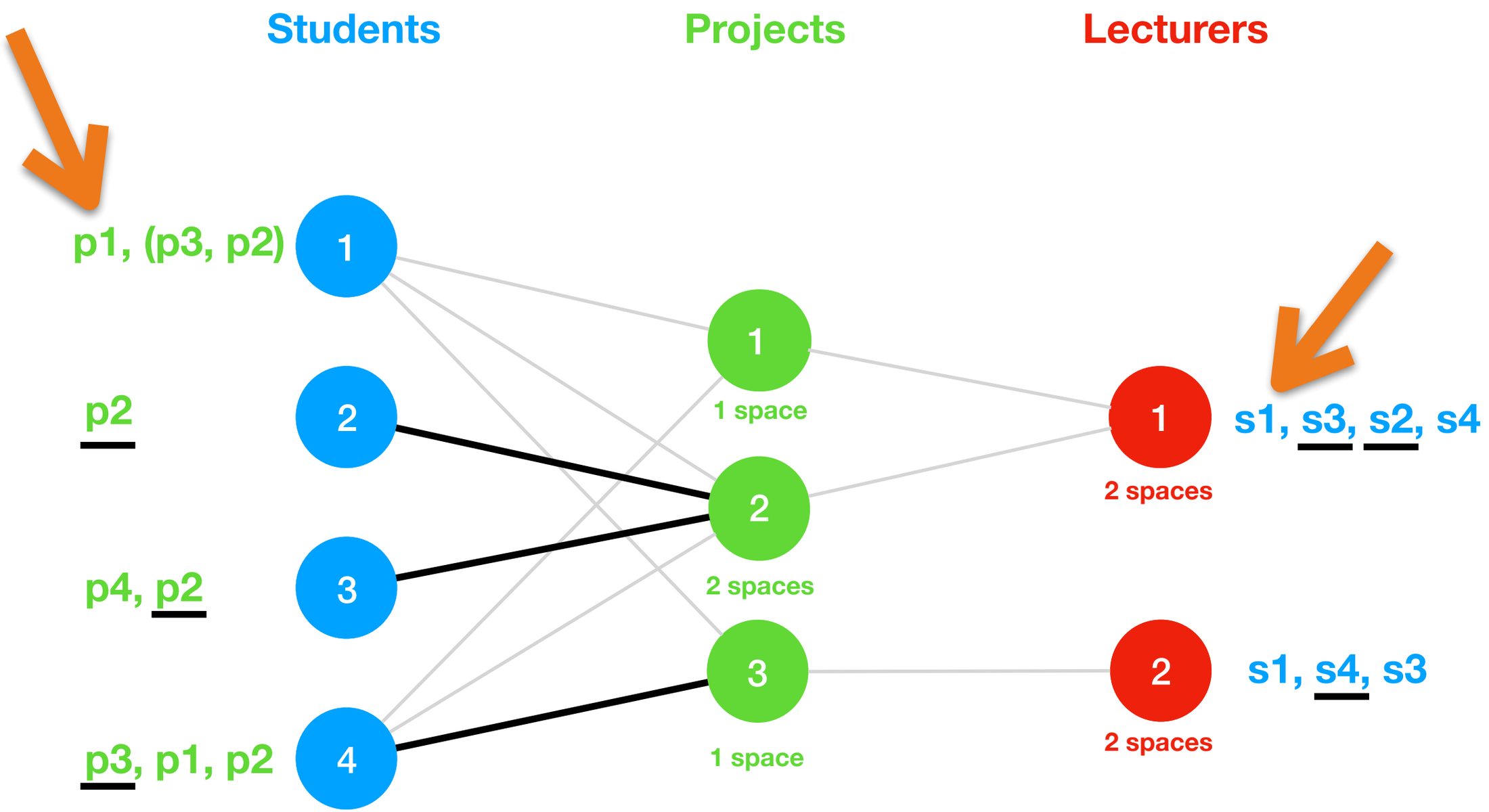


project undersubscribed, lecturer full



project full

Student-project allocation problem



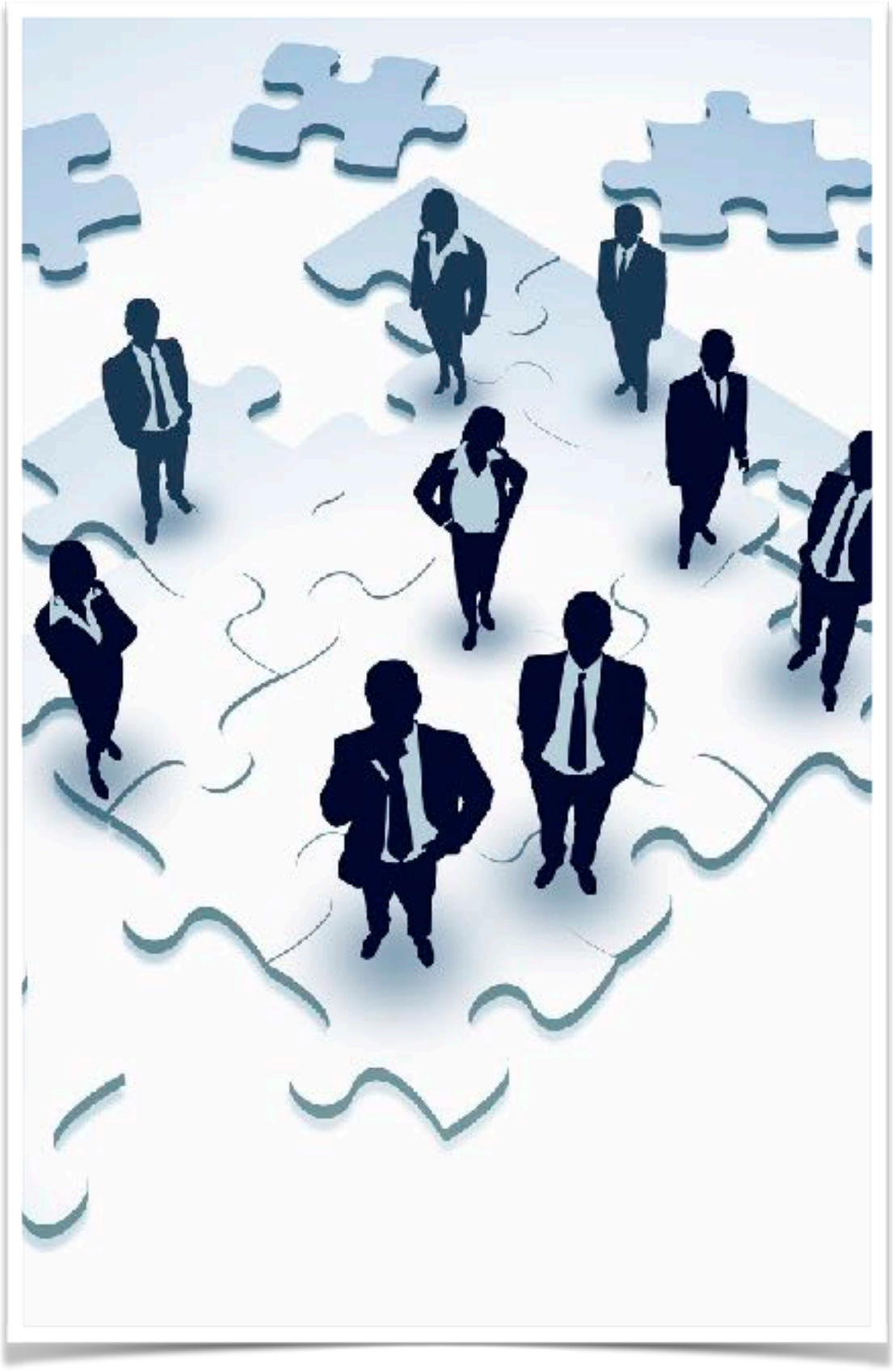
Stable matchings

- A **stable matching** is a matching with no blocking pairs
- When there are no ties in preference lists we can find a stable matching using an **efficient algorithm**. Also all stable matchings are the **same size**.
- When there are **ties in preference lists** we can find a stable matching using an **efficient algorithm** - but stable matchings are **different sizes**

Two Algorithms for the Student Project Allocation Problem; Journal of Discrete Algorithms; 2007; Abraham, Irving, Manlove

Gives us new questions

Maximum sized stable matching



Maximum sized stable matchings

Finding a maximum sized stable matching is **NP-hard**. What can we do?

- Build an exhaustive search algorithm **inefficient**
- Find an efficient algorithm for an NP-hard problem **tricky** **$1,000,000 prize**
- Integer programming (uses **inefficient** algorithms)
- Approximation algorithms (poly time)

Integer Programming

- Uses software that can optimise or solve a problem when it is given an Integer Programming model as input
- Uses exponential-time algorithms 
- But this is only in a worst-case scenario. Integer Programming algorithms are optimised to work quickly in many cases 
- An Integer programming model has been built to find a **maximum stable matching**

Approximation algorithm

- Instead of finding a **maximum stable matching**
- Look for a stable matching that is at least x% of the size of the **maximum stable matching**

- A trade off

negative: aren't solving to optimality

positive: **efficient** algorithm

Creating approximation algorithm

- An approximation algorithm exists for a simpler problem where lecturers aren't involved

Linear Time Local Approximation Algorithm for Maximum Stable Marriage; Algorithms; 2013; Kiraly

  - Can I just convert my problem and use this? No!

- Had to create a new 3/2 approximation algorithm

  - Lecturers added a lot of complications

  - Proved that this algorithm is efficient (polynomial-time) and correct (results in a stable matching at least 2/3 the size of a **maximum stable matching**)

Approximation algorithm high-level look

Students (who are not already assigned) apply in turn to their favourite project on their preference list. Assume student **s** applies to project **p**.

- if **p** and **l** (the lecturer of **p**) are undersubscribed then we add (**s**,**p**) to our matching

- if either **p** or **l** are full then we need to check whether (**s**,**p**) should replace an *existing* pair in the matching

- if there is no chance for **s** to assign to **p** then **s** will remove **p** from their preference list (and will now apply to their next favourite)

- Students iterate twice through their preference list

How good is the approximation algorithm?

- Experiments! 100s of thousands of instances with varying parameters. Ran on approximation algorithm and integer program.

- Correctness testing

- Does the approximation algorithm stick to 2/3 the size of optimal? Or do we get close to maximum?

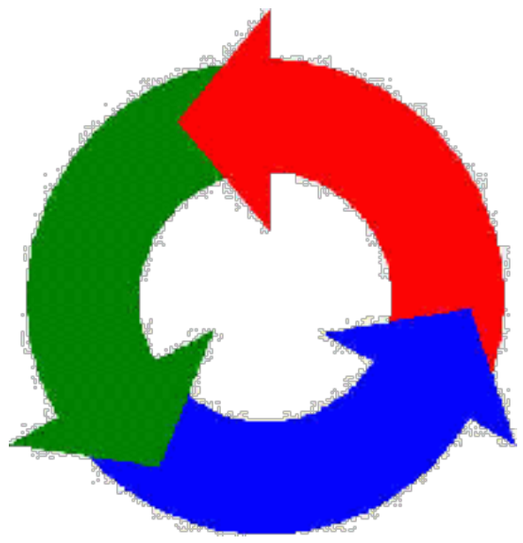
- Somewhere in between, but closer to maximum

- Much faster than using the integer program

- So is it worth using?

Future Work: coalitions

- group of several students and lecturers
- permute their assignments
- some or all get a better outcome



Summary

- Algorithms
- Student-project allocation problem
  - Finding a maximum stable matching
    - Integer programming
    - Approximation algorithm

Thank you

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