



University
of Glasgow

Regret-equality in Stable Marriage

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Joint work with: Prof David Manlove

Outline

Outline

- Matching problems

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- Matching problems
- Fairness

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- Matching problems
- Fairness
- Finding fair stable matchings

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- Finding fair stable matchings
- Experiments

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- Matching problems
- Fairness
- Finding fair stable matchings
- Experiments
- Future work

Matching Problems

Matching Problems



Matching Problems



Matching Problems



- Assign one set of entities to another set of entities

Matching Problems



- Assign one set of entities to another set of entities
- Based on preferences and capacities

Stable Marriage

Stable Marriage

Men



Women



Stable Marriage

Men

m_1

m_2

m_3

m_4

Women

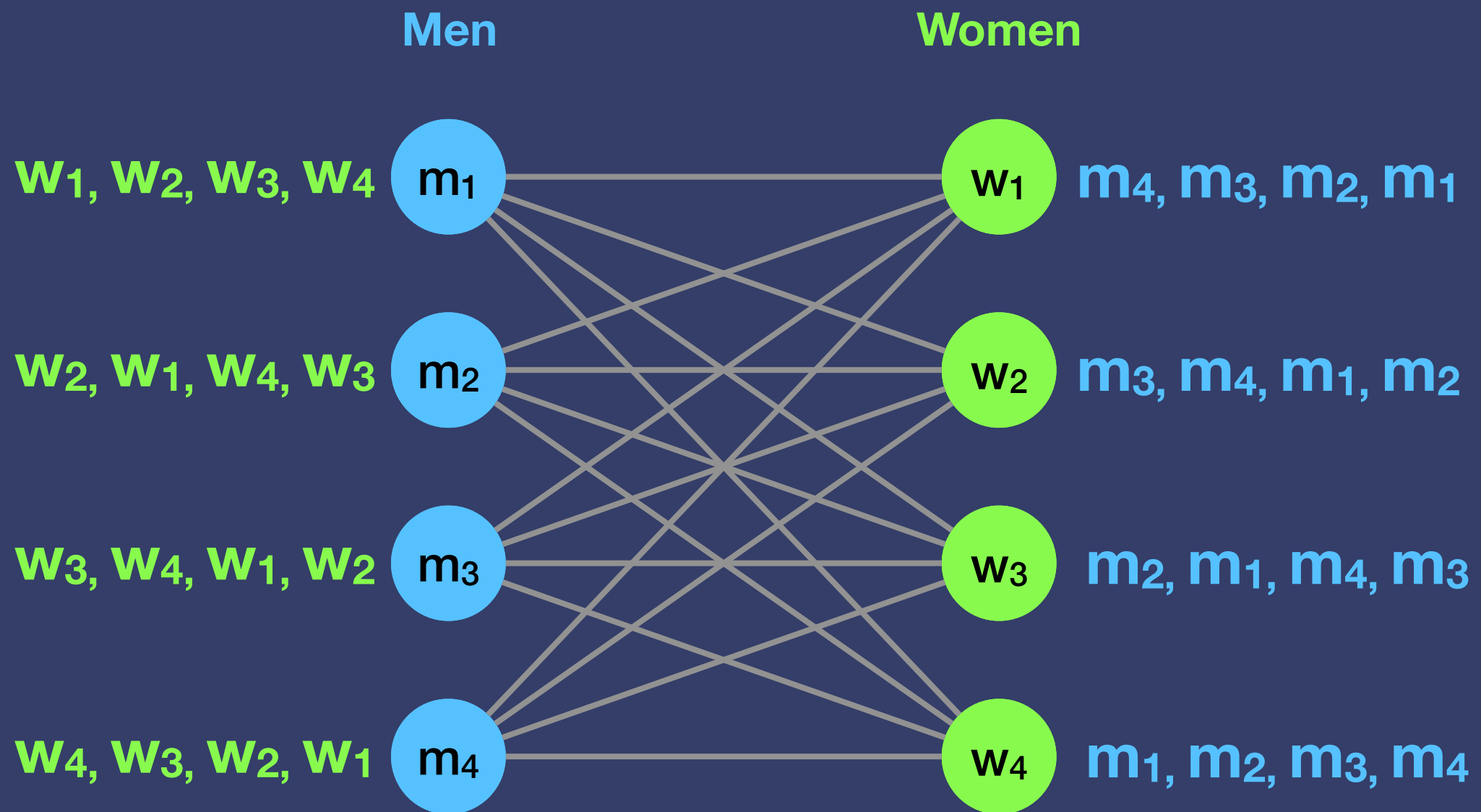
w_1

w_2

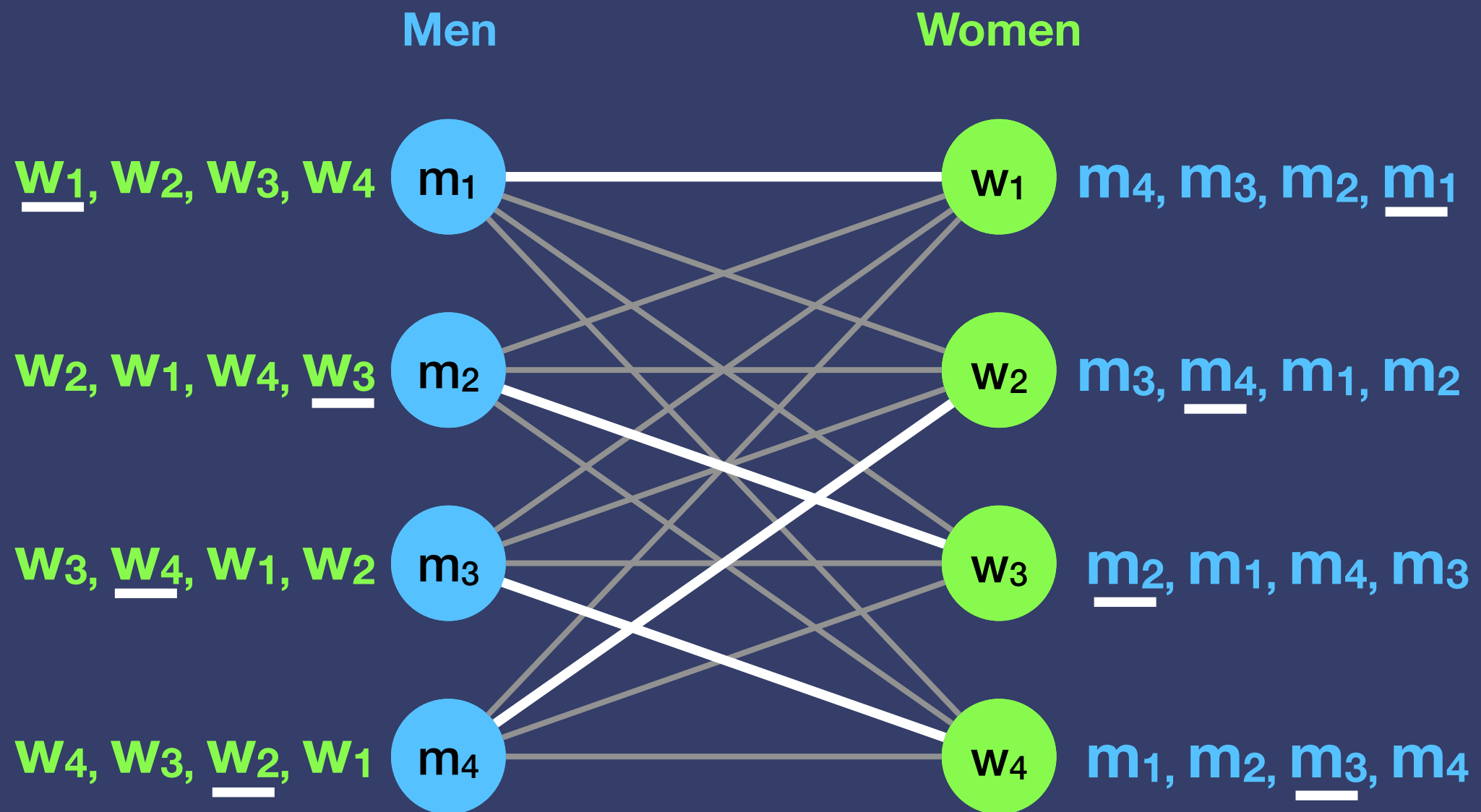
w_3

w_4

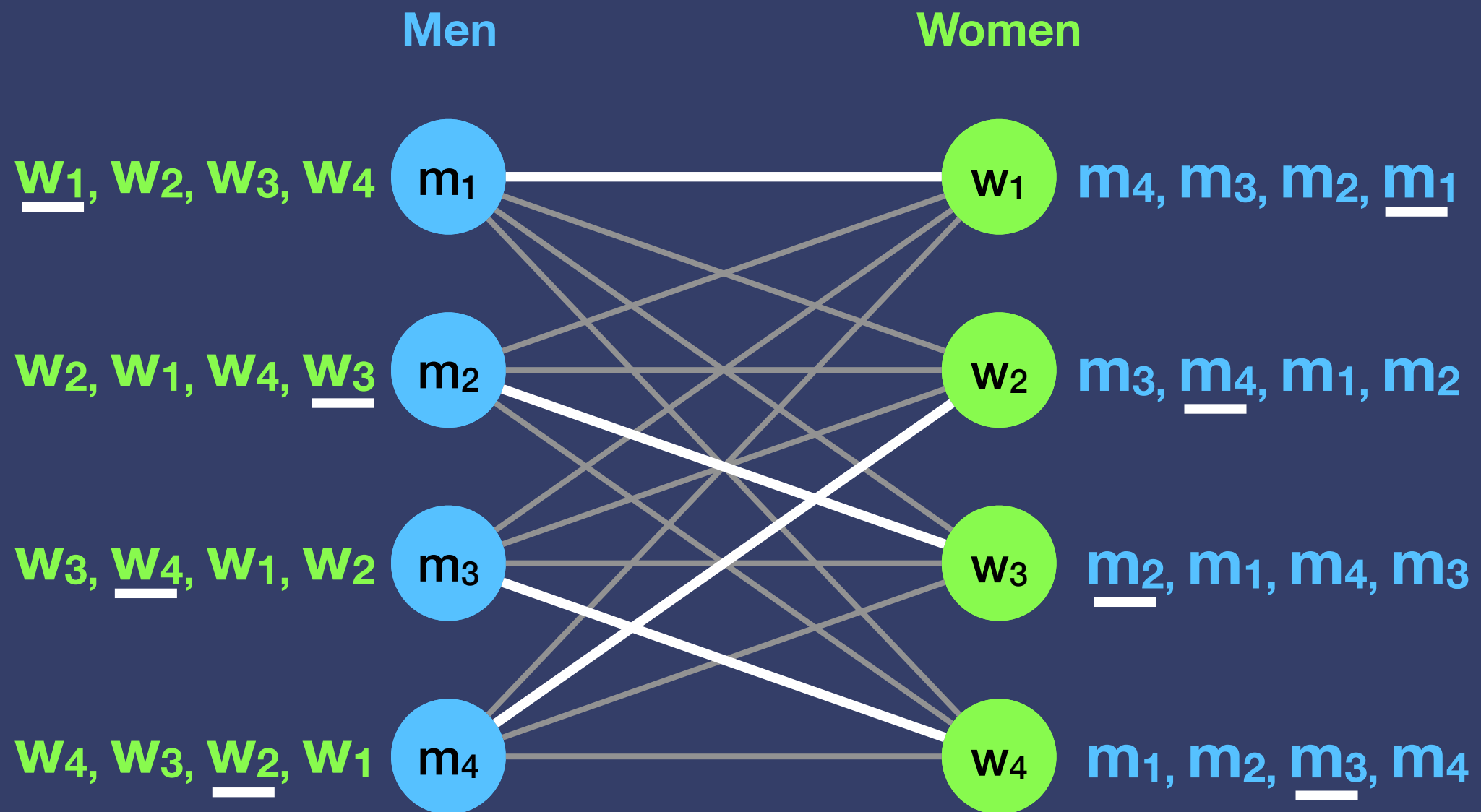
Stable Marriage



Stable Marriage



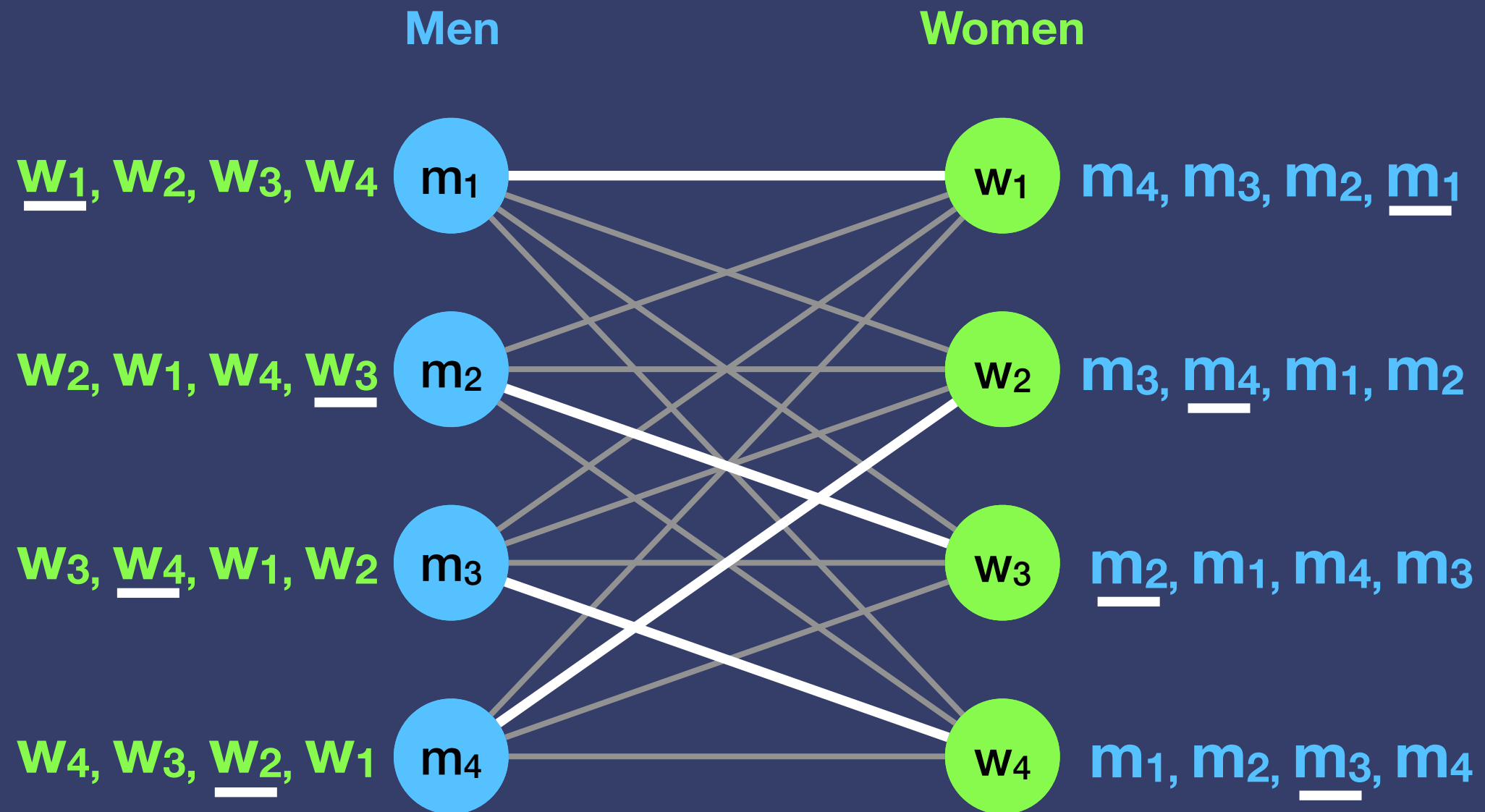
Stable Marriage



Rank

Stable Marriage

Cost: $c_U(M) = 10$, $c_W(M) = 10$

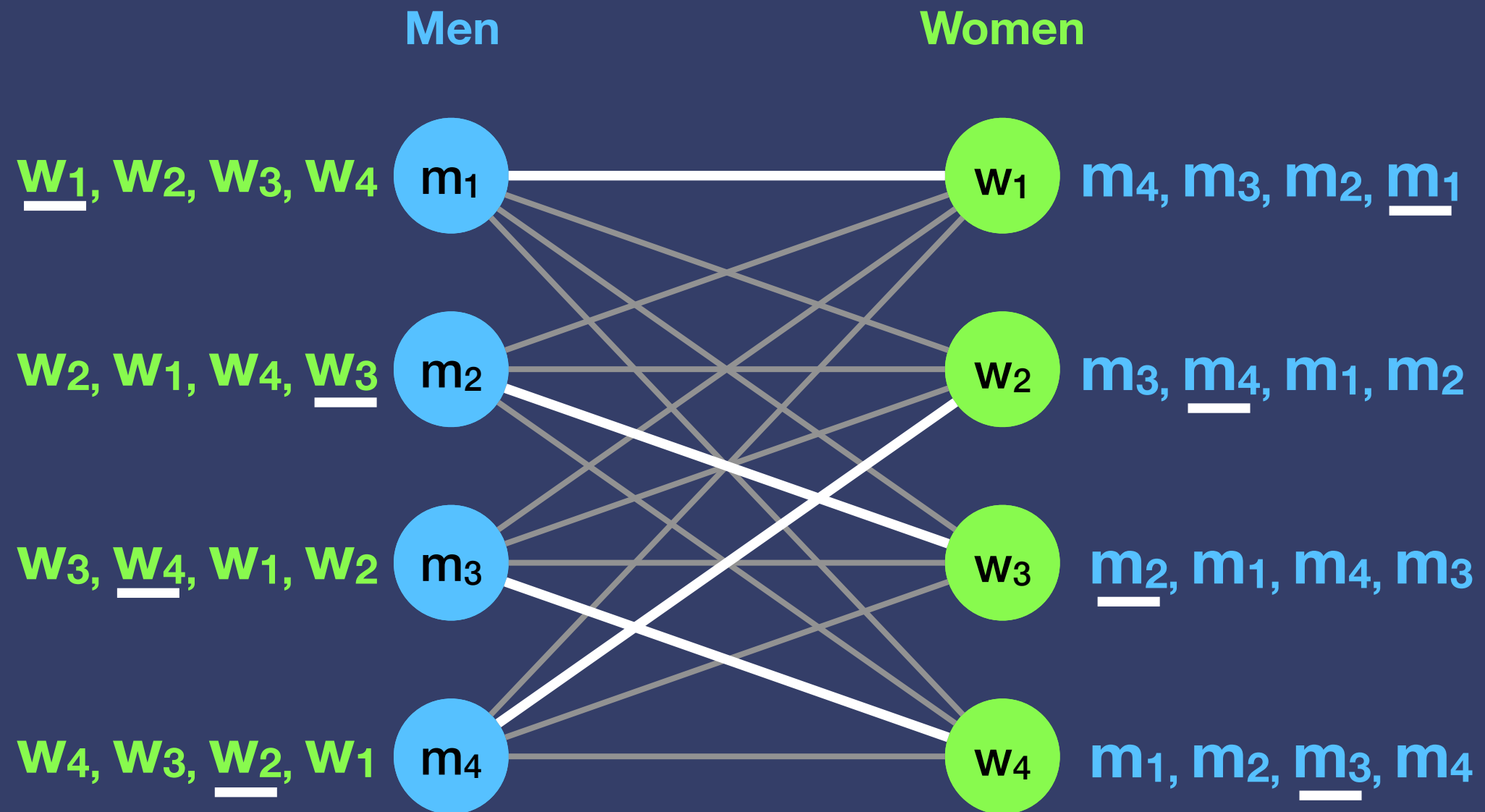


Rank

Stable Marriage

Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$



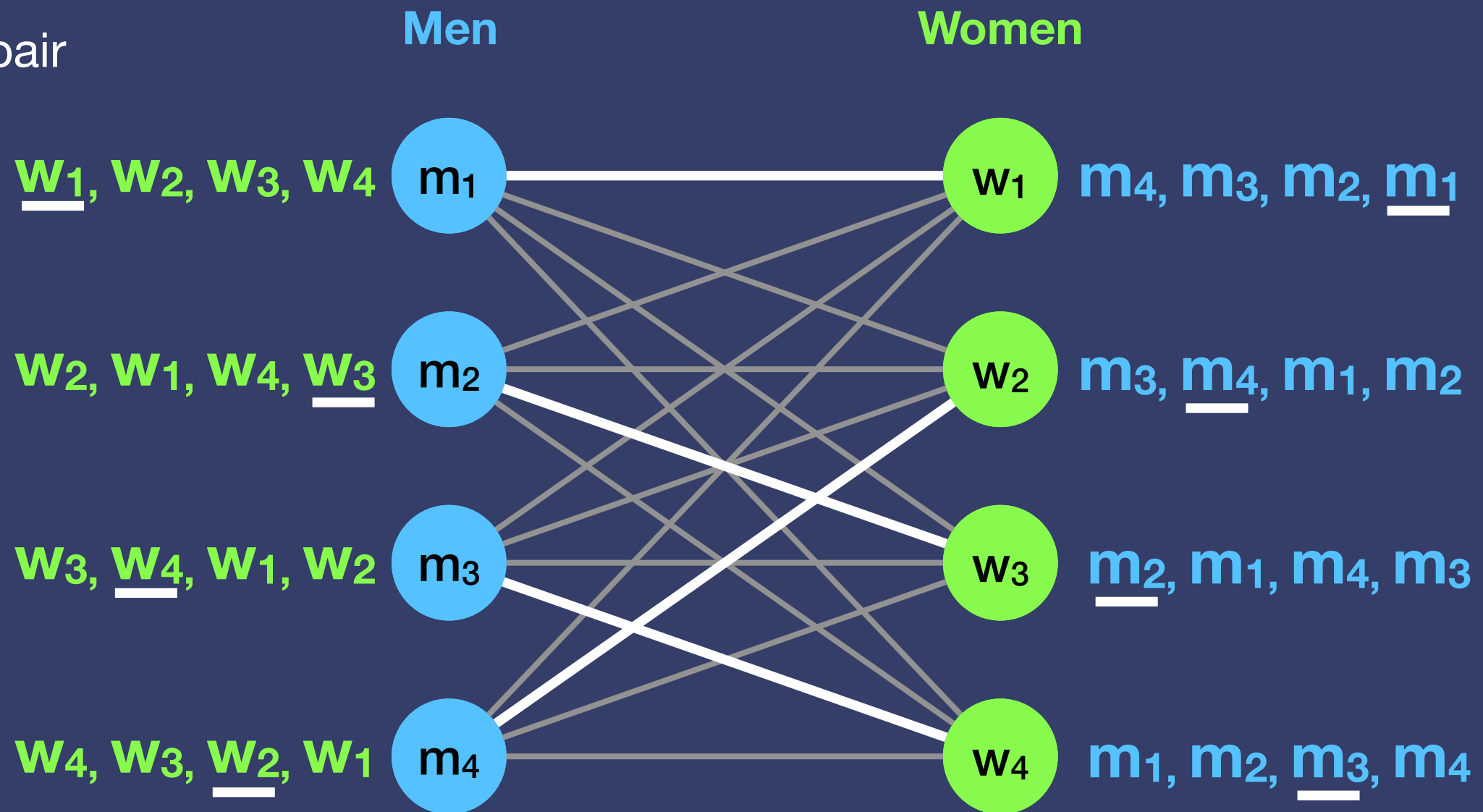
Rank

Stable Marriage

Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$

Blocking pair



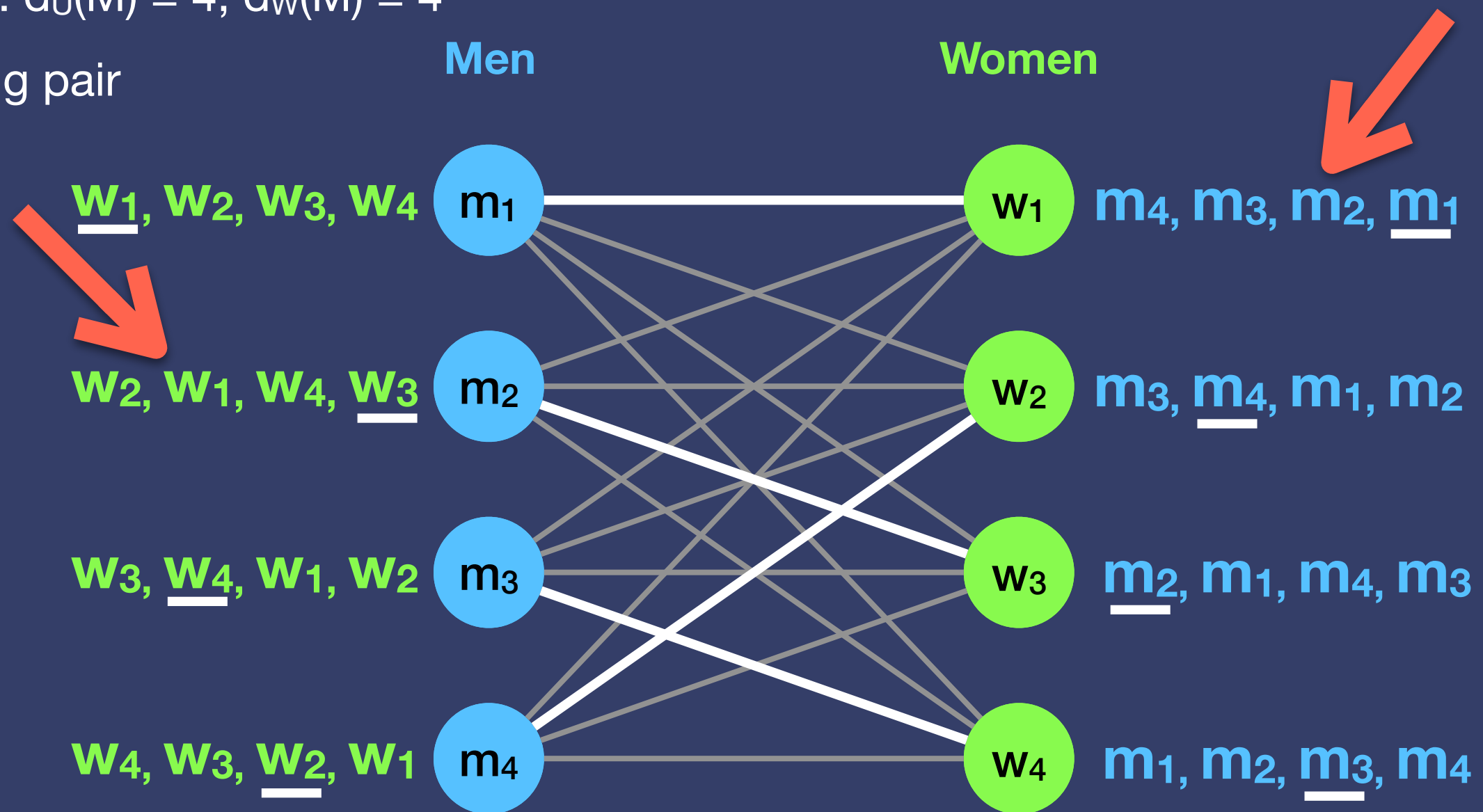
Rank

Stable Marriage

Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$

Blocking pair



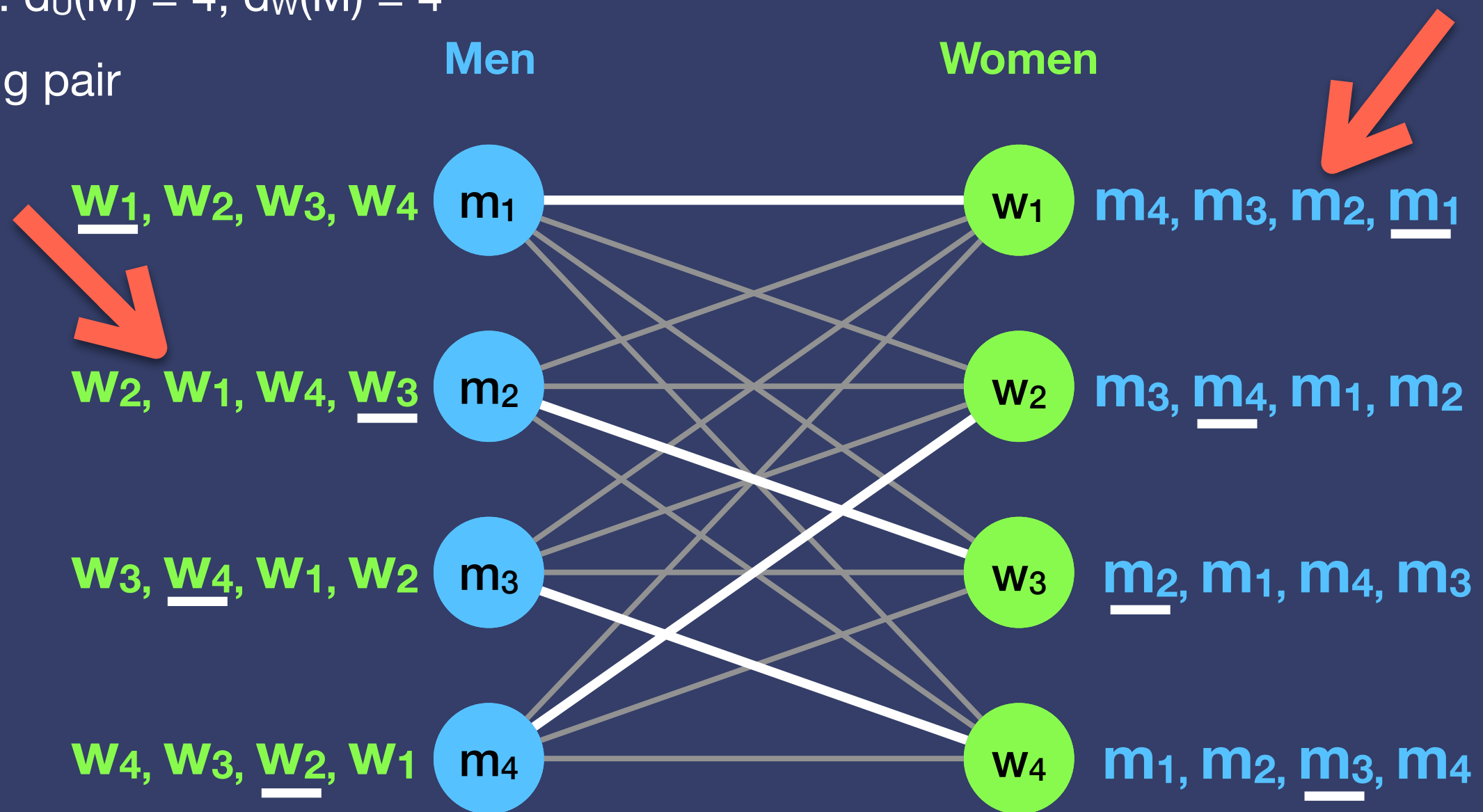
Rank

Stable Marriage

Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$

Blocking pair



A **stable matching** is a matching with no blocking pairs

Stable Marriage

Stable Marriage

- A **stable matching** is a matching with no blocking pairs

Stable Marriage

- A **stable matching** is a matching with no blocking pairs
- Many stable matchings per instance

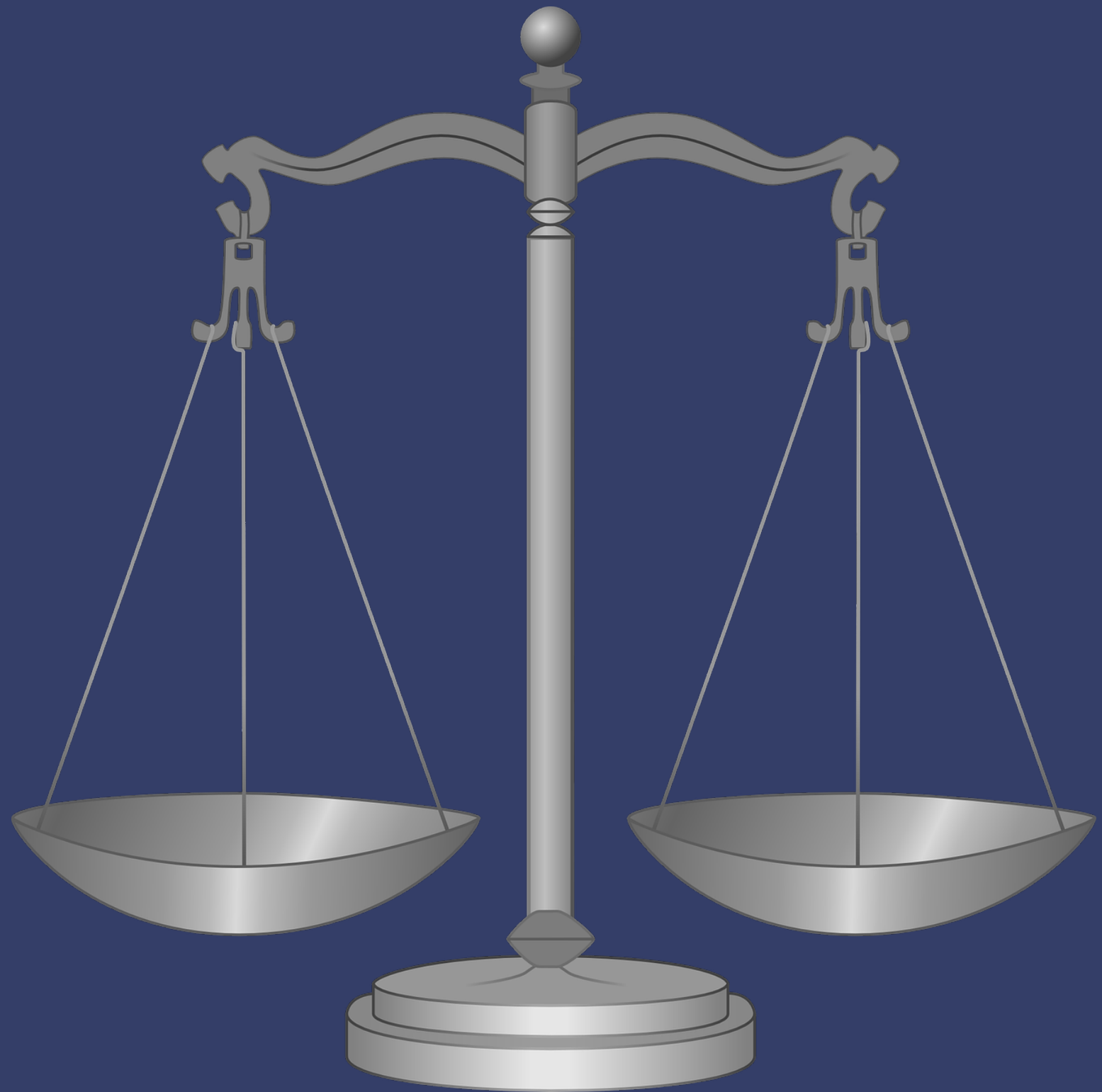
Stable Marriage

- A **stable matching** is a matching with no blocking pairs
- Many stable matchings per instance
- We can find a stable matching in linear time using the man-oriented or woman-oriented Gale-Shapley Algorithm. $O(m)$ time where m is total length of preference lists

Stable Marriage

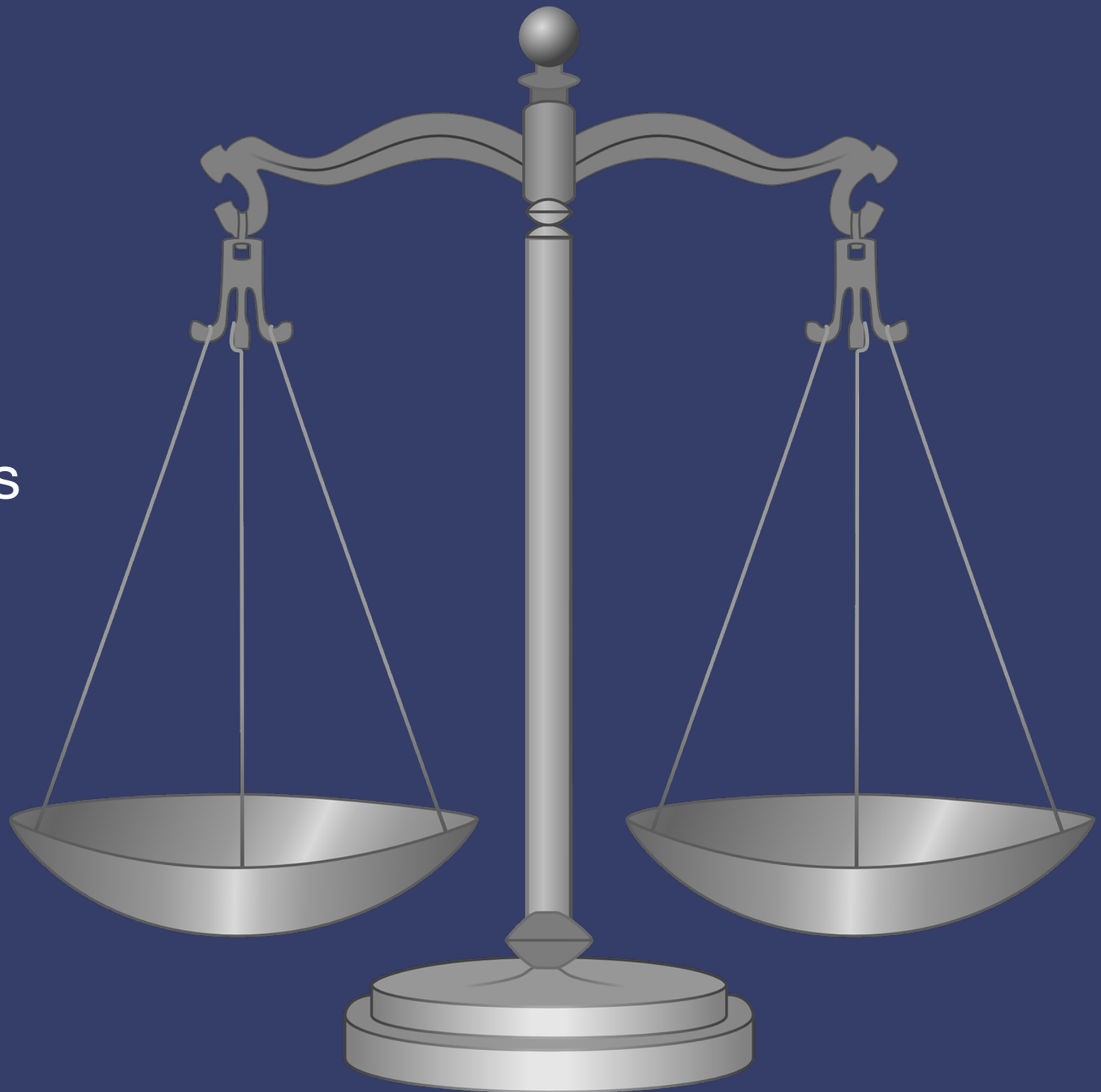
- A **stable matching** is a matching with no blocking pairs
- Many stable matchings per instance
- We can find a stable matching in linear time using the man-oriented or woman-oriented Gale-Shapley Algorithm. $O(m)$ time where m is total length of preference lists
- Man-oriented Gale-Shapley Algorithm: finds a man-optimal (woman-pessimal) stable matching (and vice versa)

Fairness



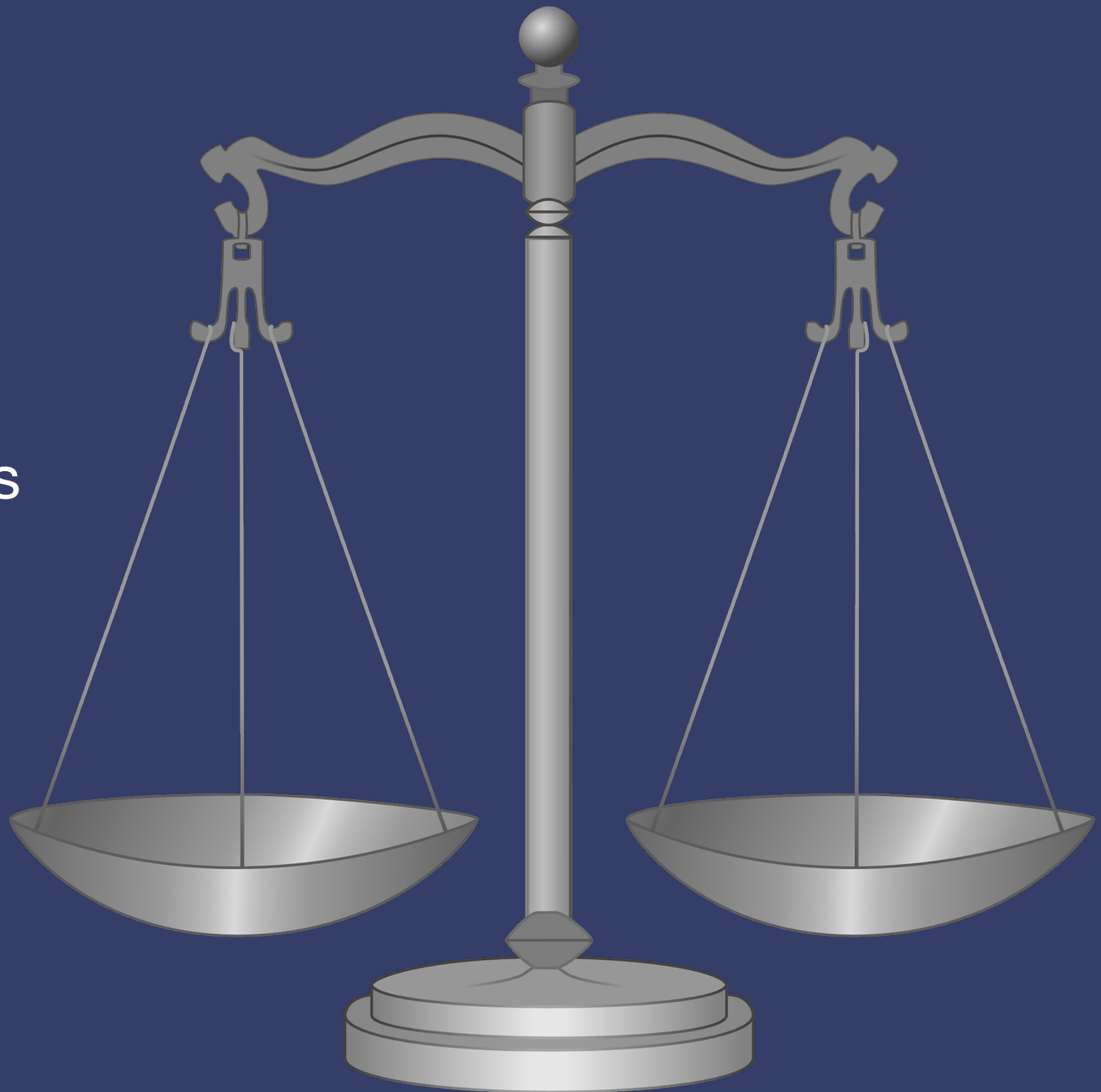
Fairness

- Want to find a stable matching that provides some kind of equality between men and women



Fairness

- Want to find a stable matching that provides some kind of equality between men and women
- Several different fairness measures



Fairness measures

Fairness measures

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum

Minimises the difference

Minimises the sum

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the
maximum

balanced score

Balanced stable
matching NP-hard

Minimises the
difference

Minimises the
sum

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the
maximum

balanced score

Balanced stable
matching NP-hard

Minimises the
difference

sex-equal score

Sex-equal stable
matching NP-hard

Minimises the
sum

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum

balanced score

Balanced stable matching NP-hard

Minimises the difference

sex-equal score

Sex-equal stable matching NP-hard

Minimises the sum

egalitarian cost

Egalitarian stable matching Poly

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum

balanced score

Balanced stable matching NP-hard

degree

Minimum-regret stable matching Poly

Minimises the difference

sex-equal score

Sex-equal stable matching NP-hard

Minimises the sum

egalitarian cost

Egalitarian stable matching Poly

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum

balanced score

Balanced stable matching NP-hard

degree

Minimum-regret stable matching Poly

Minimises the difference

sex-equal score

Sex-equal stable matching NP-hard

regret-equal score

* Regret-equal stable matching ?

Minimises the sum

egalitarian cost

Egalitarian stable matching Poly

Fairness measures

Among all stable matchings, find the stable matching that...

Cost: $c_U(M)$, $c_W(M)$

Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum

balanced score

Balanced stable matching NP-hard

degree

Minimum-regret stable matching Poly

Minimises the difference

sex-equal score

Sex-equal stable matching NP-hard

regret-equal score

* Regret-equal stable matching ?

Minimises the sum

egalitarian cost

Egalitarian stable matching Poly

regret sum score

* Min-regret sum stable matching ?

Fairness measures (degree based)

Fairness measures (degree based)

10 stable matchings for this instance

Fairness measures (degree based)

10 stable matchings for this instance

$m_1: w_1, w_2, \underline{w_3}, w_4$	$w_1: m_4, m_3, \underline{m_2}, m_1$
$m_2: w_2, \underline{w_1}, w_4, w_3$	$w_2: m_3, \underline{m_4}, m_2, m_1$
$m_3: w_3, \underline{w_4}, w_1, w_2$	$w_3: m_2, \underline{m_1}, m_4, m_3$
$m_4: w_4, w_3, \underline{w_2}, w_1$	$w_4: m_1, m_2, \underline{m_3}, m_4$

Fairness measures (degree based)

10 stable matchings for this instance

$m_1: w_1, w_2, \underline{w_3}, w_4$	$w_1: m_4, \underline{m_3}, \underline{m_2}, m_1$
$m_2: w_2, \underline{w_1}, w_4, w_3$	$w_2: m_3, \underline{m_4}, m_2, m_1$
$m_3: w_3, \underline{w_4}, w_1, w_2$	$w_3: m_2, \underline{m_1}, m_4, m_3$
$m_4: w_4, w_3, \underline{w_2}, w_1$	$w_4: m_1, m_2, \underline{m_3}, m_4$

$m_1: w_1, w_2, \underline{w_3}, w_4$	$w_1: m_4, \underline{m_3}, m_2, m_1$
$m_2: w_2, w_1, \underline{w_4}, w_3$	$w_2: m_3, \underline{m_4}, m_2, m_1$
$m_3: w_3, w_4, \underline{w_1}, w_2$	$w_3: m_2, \underline{m_1}, m_4, m_3$
$m_4: w_4, w_3, \underline{w_2}, w_1$	$w_4: m_1, \underline{m_2}, m_3, m_4$

Fairness measures (degree based)

10 stable matchings for this instance

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, m_3, \underline{m_2}, m_1$
m_2 : $w_2, \underline{w_1}, w_4, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, \underline{w_4}, w_1, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, m_2, \underline{m_3}, m_4$

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, \underline{m_3}, m_2, m_1$
m_2 : $w_2, w_1, \underline{w_4}, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, w_4, \underline{w_1}, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, \underline{m_2}, m_3, m_4$

m_1 : $\underline{w_1}, w_2, w_3, w_4$	w_1 : $m_4, m_3, m_2, \underline{m_1}$
m_2 : $\underline{w_2}, w_1, w_4, w_3$	w_2 : $m_3, m_4, \underline{m_2}, m_1$
m_3 : $\underline{w_3}, w_4, w_1, w_2$	w_3 : $m_2, m_1, m_4, \underline{m_3}$
m_4 : $\underline{w_4}, w_3, w_2, w_1$	w_4 : $m_1, m_2, m_3, \underline{m_4}$

Fairness measures (degree based)

10 stable matchings for this instance

m₁: w₁ , w₂ , <u>w₃</u> , w₄	w₁: m₄ , m₃ , <u>m₂</u> , m₁
m₂: w₂ , <u>w₁</u> , w₄ , w₃	w₂: m₃ , <u>m₄</u> , m₂ , m₁
m₃: w₃ , <u>w₄</u> , w₁ , w₂	w₃: m₂ , <u>m₁</u> , m₄ , m₃
m₄: w₄ , w₃ , <u>w₂</u> , w₁	w₄: m₁ , m₂ , <u>m₃</u> , m₄

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

m₁: w₁ , w₂ , <u>w₃</u> , w₄	w₁: m₄ , <u>m₃</u> , m₂ , m₁
m₂: w₂ , w₁ , <u>w₄</u> , w₃	w₂: m₃ , <u>m₄</u> , m₂ , m₁
m₃: w₃ , w₄ , <u>w₁</u> , w₂	w₃: m₂ , <u>m₁</u> , m₄ , m₃
m₄: w₄ , w₃ , <u>w₂</u> , w₁	w₄: m₁ , <u>m₂</u> , m₃ , m₄

m₁: <u>w₁</u> , w₂ , w₃ , w₄	w₁: m₄ , m₃ , m₂ , <u>m₁</u>
m₂: <u>w₂</u> , w₁ , w₄ , w₃	w₂: m₃ , m₄ , <u>m₂</u> , m₁
m₃: <u>w₃</u> , w₄ , w₁ , w₂	w₃: m₂ , m₁ , m₄ , <u>m₃</u>
m₄: <u>w₄</u> , w₃ , w₂ , w₁	w₄: m₁ , m₂ , m₃ , <u>m₄</u>

Fairness measures (degree based)

10 stable matchings for this instance

m₁: w₁ , w₂ , <u>w₃</u> , w₄	w₁: m₄ , m₃ , <u>m₂</u> , m₁
m₂: w₂ , <u>w₁</u> , w₄ , w₃	w₂: m₃ , <u>m₄</u> , m₂ , m₁
m₃: w₃ , <u>w₄</u> , w₁ , w₂	w₃: m₂ , <u>m₁</u> , m₄ , m₃
m₄: w₄ , w₃ , <u>w₂</u> , w₁	w₄: m₁ , m₂ , <u>m₃</u> , m₄

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

m₁: w₁ , w₂ , <u>w₃</u> , w₄	w₁: m₄ , <u>m₃</u> , m₂ , m₁
m₂: w₂ , w₁ , <u>w₄</u> , w₃	w₂: m₃ , <u>m₄</u> , m₂ , m₁
m₃: w₃ , w₄ , <u>w₁</u> , w₂	w₃: m₂ , <u>m₁</u> , m₄ , m₃
m₄: w₄ , w₃ , <u>w₂</u> , w₁	w₄: m₁ , <u>m₂</u> , m₃ , m₄

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

m₁: <u>w₁</u> , w₂ , w₃ , w₄	w₁: m₄ , m₃ , m₂ , <u>m₁</u>
m₂: <u>w₂</u> , w₁ , w₄ , w₃	w₂: m₃ , m₄ , <u>m₂</u> , m₁
m₃: <u>w₃</u> , w₄ , w₁ , w₂	w₃: m₂ , m₁ , m₄ , <u>m₃</u>
m₄: <u>w₄</u> , w₃ , w₂ , w₁	w₄: m₁ , m₂ , m₃ , <u>m₄</u>

Fairness measures (degree based)

10 stable matchings for this instance

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, m_3, \underline{m_2}, m_1$
m_2 : $w_2, \underline{w_1}, w_4, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, \underline{w_4}, w_1, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, m_2, \underline{m_3}, m_4$

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, \underline{m_3}, m_2, m_1$
m_2 : $w_2, w_1, \underline{w_4}, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, w_4, \underline{w_1}, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, \underline{m_2}, m_3, m_4$

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

m_1 : $\underline{w_1}, w_2, w_3, w_4$	w_1 : $m_4, m_3, m_2, \underline{m_1}$
m_2 : $\underline{w_2}, w_1, w_4, w_3$	w_2 : $m_3, m_4, \underline{m_2}, m_1$
m_3 : $\underline{w_3}, w_4, w_1, w_2$	w_3 : $m_2, m_1, m_4, \underline{m_3}$
m_4 : $\underline{w_4}, w_3, w_2, w_1$	w_4 : $m_1, m_2, m_3, \underline{m_4}$

Degree: 4

Regret-equality score: 3

Min-regret sum score: 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1 : $w_1, w_2, \underline{w_3}, w_4$
 m_2 : $w_2, \underline{w_1}, w_4, w_3$
 m_3 : $w_3, \underline{w_4}, w_1, w_2$
 m_4 : $w_4, w_3, \underline{w_2}, w_1$

w_1 : $m_4, m_3, \underline{m_2}, m_1$
 w_2 : $m_3, \underline{m_4}, m_2, m_1$
 w_3 : $m_2, \underline{m_1}, m_4, m_3$
 w_4 : $m_1, m_2, \underline{m_3}, m_4$

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

m_1 : $w_1, w_2, \underline{w_3}, w_4$
 m_2 : $w_2, w_1, \underline{w_4}, w_3$
 m_3 : $w_3, w_4, \underline{w_1}, w_2$
 m_4 : $w_4, w_3, \underline{w_2}, w_1$

w_1 : $m_4, m_3, m_2, \underline{m_1}$
 w_2 : $m_3, \underline{m_4}, m_2, m_1$
 w_3 : $m_2, \underline{m_1}, m_4, m_3$
 w_4 : $m_1, \underline{m_2}, m_3, m_4$

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

m_1 : $\underline{w_1}, w_2, w_3, w_4$
 m_2 : $\underline{w_2}, w_1, w_4, w_3$
 m_3 : $\underline{w_3}, w_4, w_1, w_2$
 m_4 : $\underline{w_4}, w_3, w_2, w_1$

w_1 : $m_4, m_3, m_2, \underline{m_1}$
 w_2 : $m_3, m_4, \underline{m_2}, m_1$
 w_3 : $m_2, m_1, m_4, \underline{m_3}$
 w_4 : $m_1, m_2, m_3, \underline{m_4}$

Degree: 4

Regret-equality score: 3

Min-regret sum score: 5

Over all stable matchings:

Minimum degree = 3

Minimum regret-equality score = 0

Minimum regret sum score = 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, m_3, \underline{m_2}, m_1$
m_2 : $w_2, \underline{w_1}, w_4, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, \underline{w_4}, w_1, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, m_2, \underline{m_3}, m_4$

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, \underline{m_3}, m_2, m_1$
m_2 : $w_2, w_1, \underline{w_4}, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, w_4, \underline{w_1}, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, \underline{m_2}, m_3, m_4$

m_1 : $\underline{w_1}, w_2, w_3, w_4$	w_1 : $m_4, m_3, m_2, \underline{m_1}$
m_2 : $\underline{w_2}, w_1, w_4, w_3$	w_2 : $m_3, m_4, \underline{m_2}, m_1$
m_3 : $\underline{w_3}, w_4, w_1, w_2$	w_3 : $m_2, m_1, m_4, \underline{m_3}$
m_4 : $\underline{w_4}, w_3, w_2, w_1$	w_4 : $m_1, m_2, m_3, \underline{m_4}$

Min-regret & Regret-equal

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

Degree: 4

Regret-equality score: 3

Min-regret sum score: 5

Over all stable matchings:

Minimum degree = 3

Minimum regret-equality score = 0

Minimum regret sum score = 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, m_3, \underline{m_2}, m_1$
m_2 : $w_2, \underline{w_1}, w_4, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, \underline{w_4}, w_1, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, m_2, \underline{m_3}, m_4$

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, \underline{m_3}, m_2, m_1$
m_2 : $w_2, w_1, \underline{w_4}, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, w_4, \underline{w_1}, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, \underline{m_2}, m_3, m_4$

m_1 : $\underline{w_1}, w_2, w_3, w_4$	w_1 : $m_4, m_3, m_2, \underline{m_1}$
m_2 : $\underline{w_2}, w_1, w_4, w_3$	w_2 : $m_3, m_4, \underline{m_2}, m_1$
m_3 : $\underline{w_3}, w_4, w_1, w_2$	w_3 : $m_2, m_1, m_4, \underline{m_3}$
m_4 : $\underline{w_4}, w_3, w_2, w_1$	w_4 : $m_1, m_2, m_3, \underline{m_4}$

Min-regret & Regret-equal

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

Min-regret & Min-regret sum

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

Degree: 4

Regret-equality score: 3

Min-regret sum score: 5

Over all stable matchings:

Minimum degree = 3

Minimum regret-equality score = 0

Minimum regret sum score = 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1 : $w_1, w_2, \underline{w_3}, w_4$	w_1 : $m_4, m_3, \underline{m_2}, m_1$
m_2 : $w_2, \underline{w_1}, w_4, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, \underline{w_4}, w_1, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
m_4 : $w_4, w_3, \underline{w_2}, w_1$	w_4 : $m_1, m_2, \underline{m_3}, m_4$

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m_2 : $w_2, w_1, \underline{w_4}, w_3$	w_2 : $m_3, \underline{m_4}, m_2, m_1$
m_3 : $w_3, w_4, \underline{w_1}, w_2$	w_3 : $m_2, \underline{m_1}, m_4, m_3$
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Over all stable matchings:

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Finding a Regret-Equal Stable Matching



Rotations

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- Rotation - series of man-woman pairs that take us from one stable matching to another when permuted

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R_1 m_1 m_4
 w_2 w_3

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R₁ **m₁ m₄**
 w₂ w₃

M₁ **m₁ m₂ m₃ m₄**
 w₂ w₁ w₄ w₃

Rotations

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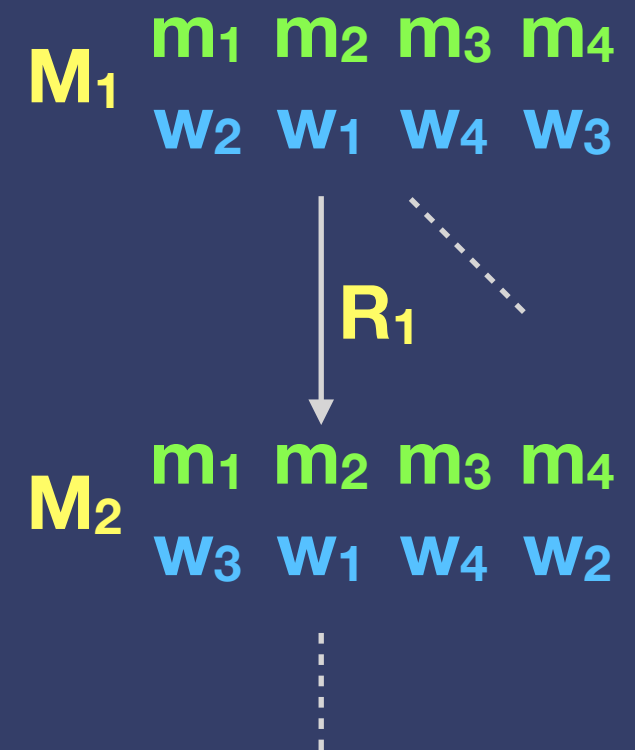


M_2 m_1 m_2 m_3 m_4
 w_3 w_1 w_4 w_2

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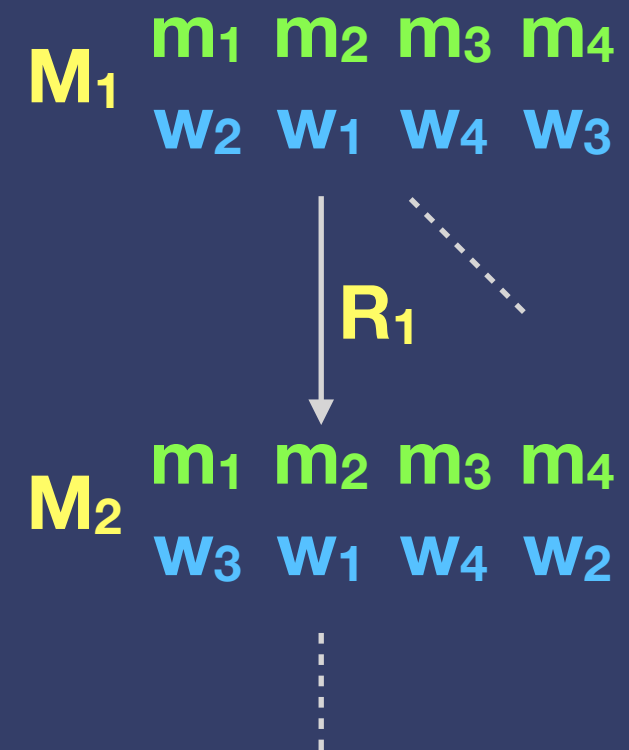


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- Can only eliminate *exposed* rotations



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M_1 m_1 m_2 m_3 m_4
 w_2 w_1 w_4 w_3

R_1

M_2 m_1 m_2 m_3 m_4
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Rotations

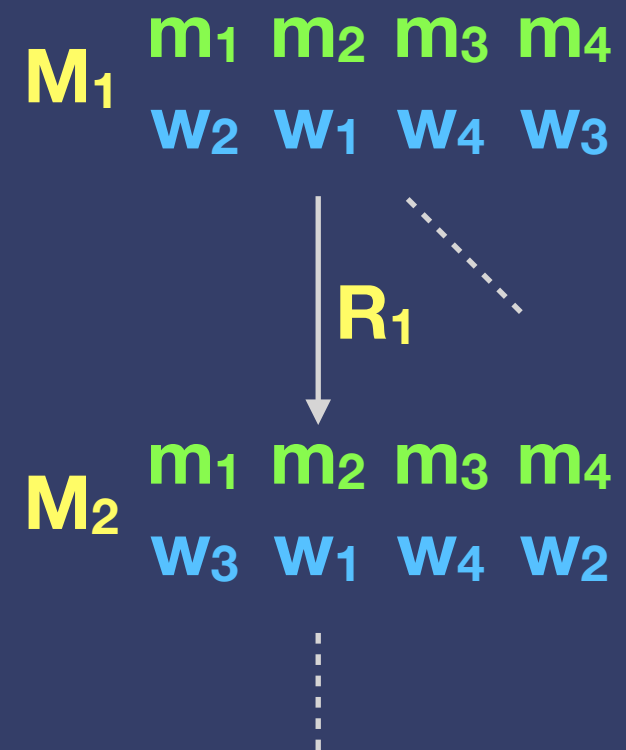
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R_1 $m_1 \ m_4$
 $w_2 \ w_3$

- Can only eliminate *exposed* rotations

R_2 $m_1 \ m_2$
 $w_1 \ w_2$

- $O(n^2)$ algorithm to find all rotations



Rotations

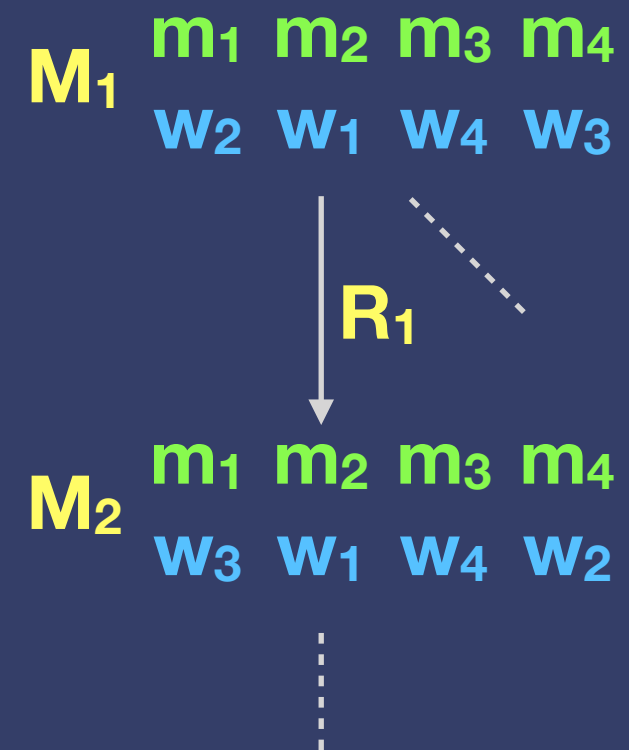
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R_1 m_1 m_4
 w_2 w_3

- Can only eliminate *exposed* rotations

R_2 m_1 m_2
 w_1 w_2

- $O(n^2)$ algorithm to find all rotations
- Rotations form a structure to allow enumeration of all stable matchings. All rotation makes some men worse off and some women better off



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Say $d_u(M_0) = 2$ and $d_w(M_0) = 5$ $d(M_0) = (2, 5)$
- Then, a regret equal stable matching must exist within the following degrees pairs:

r-e score: 3 $(2, 5)$

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r-e score: 1 $(2, 3)$ $(3, 4)$ $(4, 5)$

r-e score: 0 $(2, 2)$ $(3, 3)$ $(4, 4)$ $(5, 5)$

r-e score: 1 $(2, 1)$ $(3, 2)$ $(4, 3)$ $(5, 4)$ $(6, 5)$

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why are these the only possible degrees?

- M_0 has a r-e score of 3
- men can only get worse
- women can only get better

Algorithm

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Algorithm

2. If $d_U(M_0) \geq d_W(M_0)$ then exit with M_0
 3. For each man m and for each column c :
 1. rotate m down to c (if possible)
 2. rotate women down column c who have worst rank
- r-e score: 3 (2, 5)

r-e score: 2 (2, 4) (3, 5)

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r-e score: 0 (2, 2) (3, 3) (4, 4) (5, 5)

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- Stop iterating women up the column when $d_U(M) \geq d_W(M)$
 - Save the best matching as you go

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Time complexity

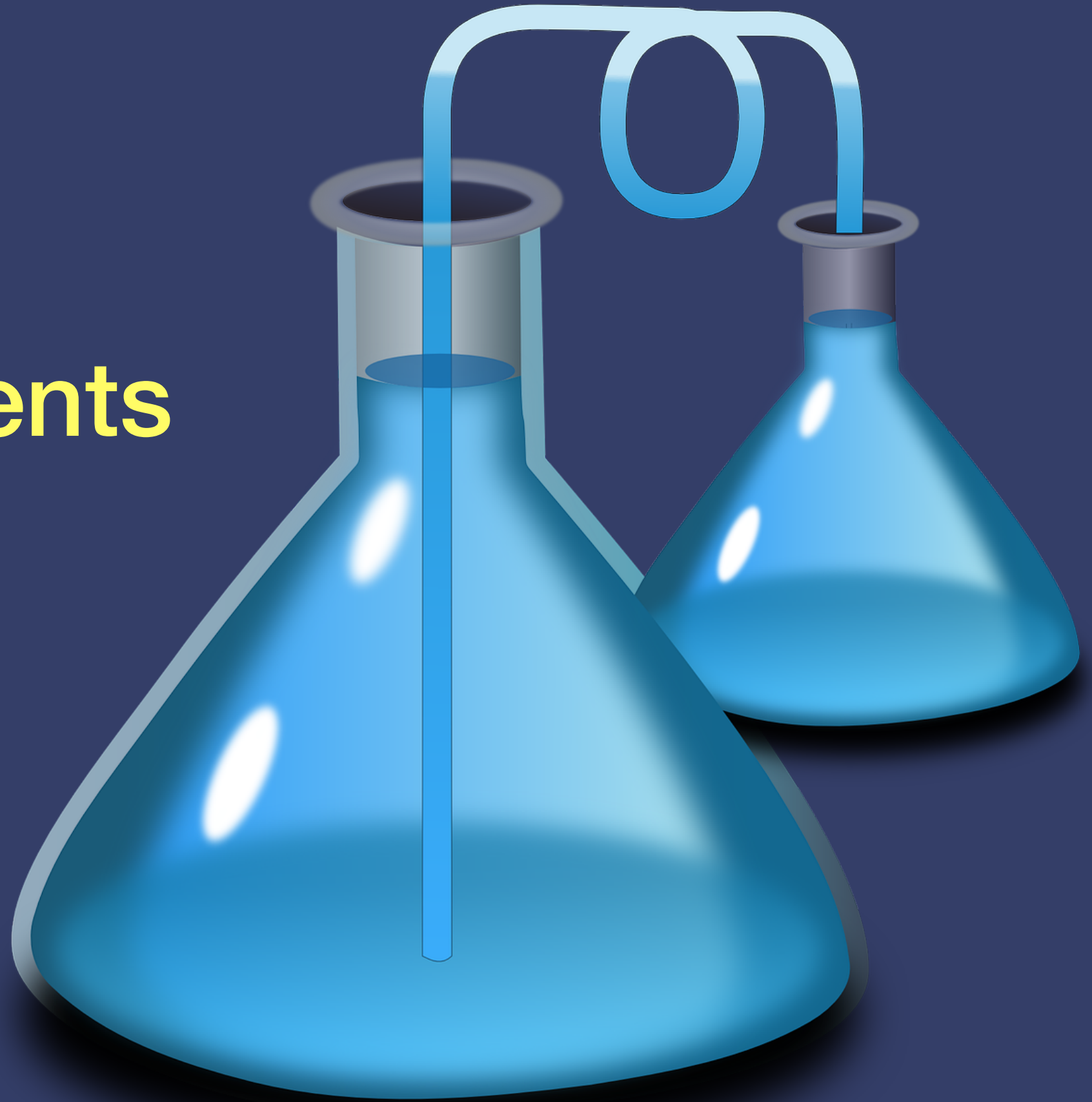
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★ Total $O(n^3c)$ ★

Experiments



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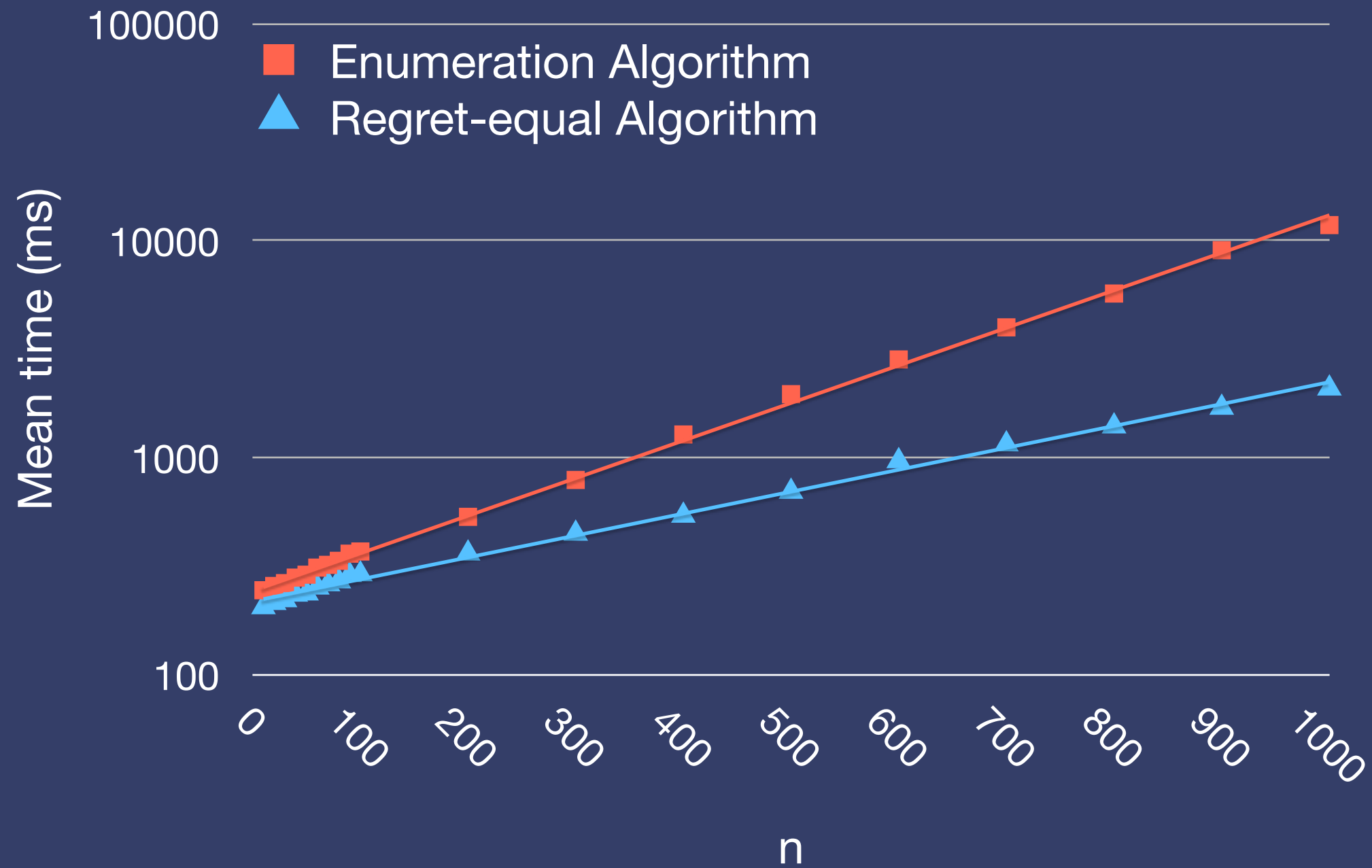
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 - CPLEX up to size $n = 50$ for the enumeration algorithm

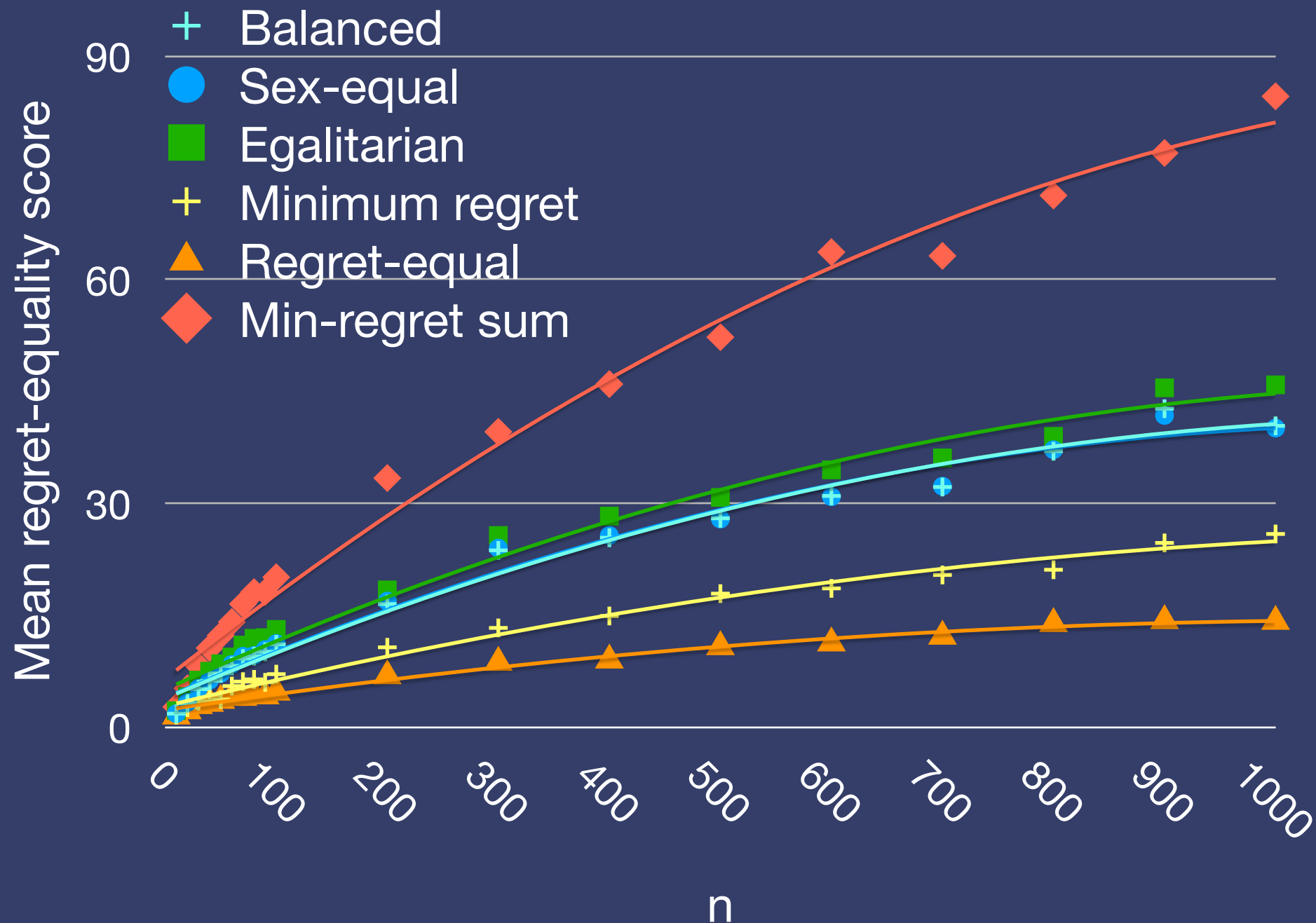
Time taken

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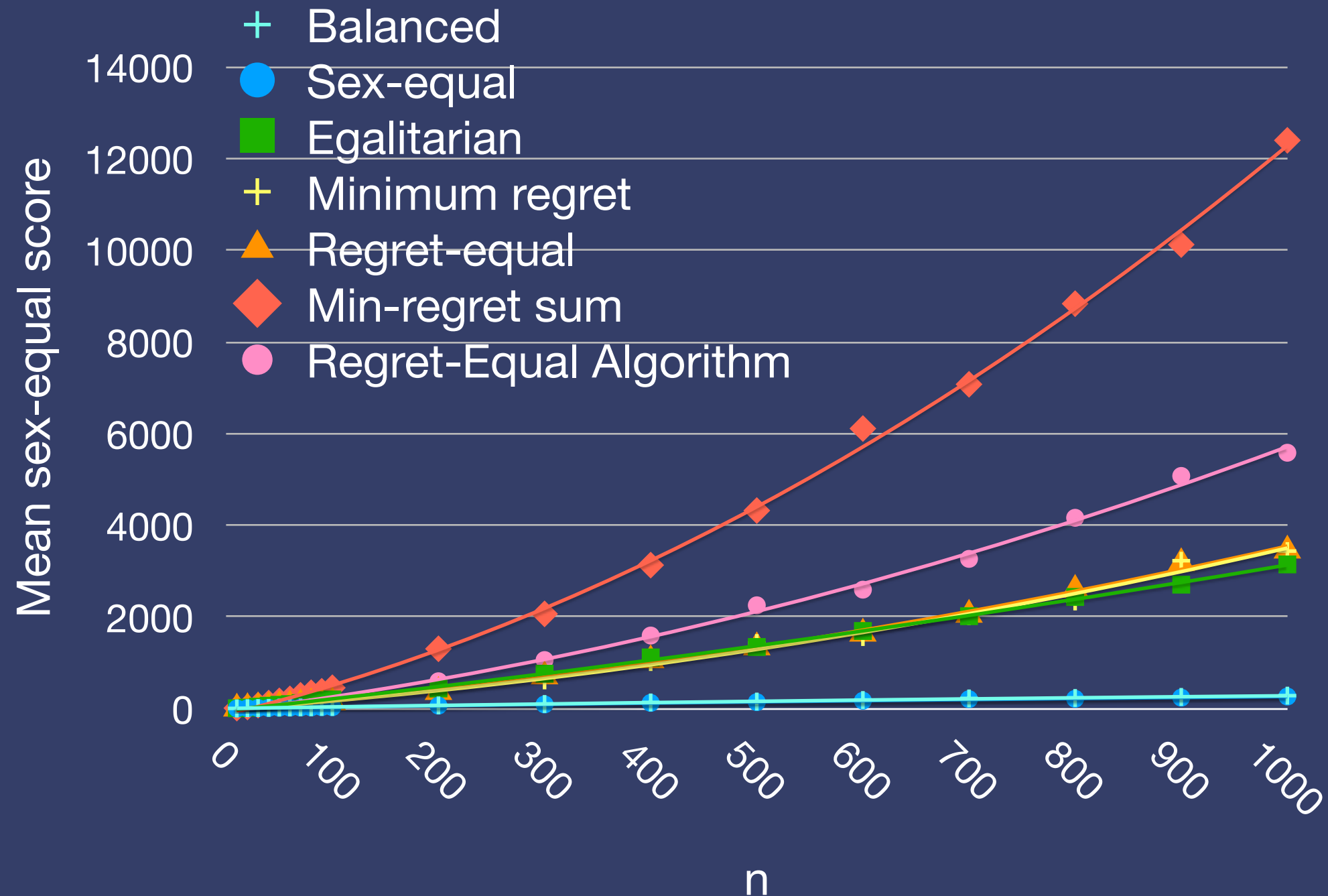
Regret-equality score for different optimal matchings

Regret-equality score for different optimal matchings



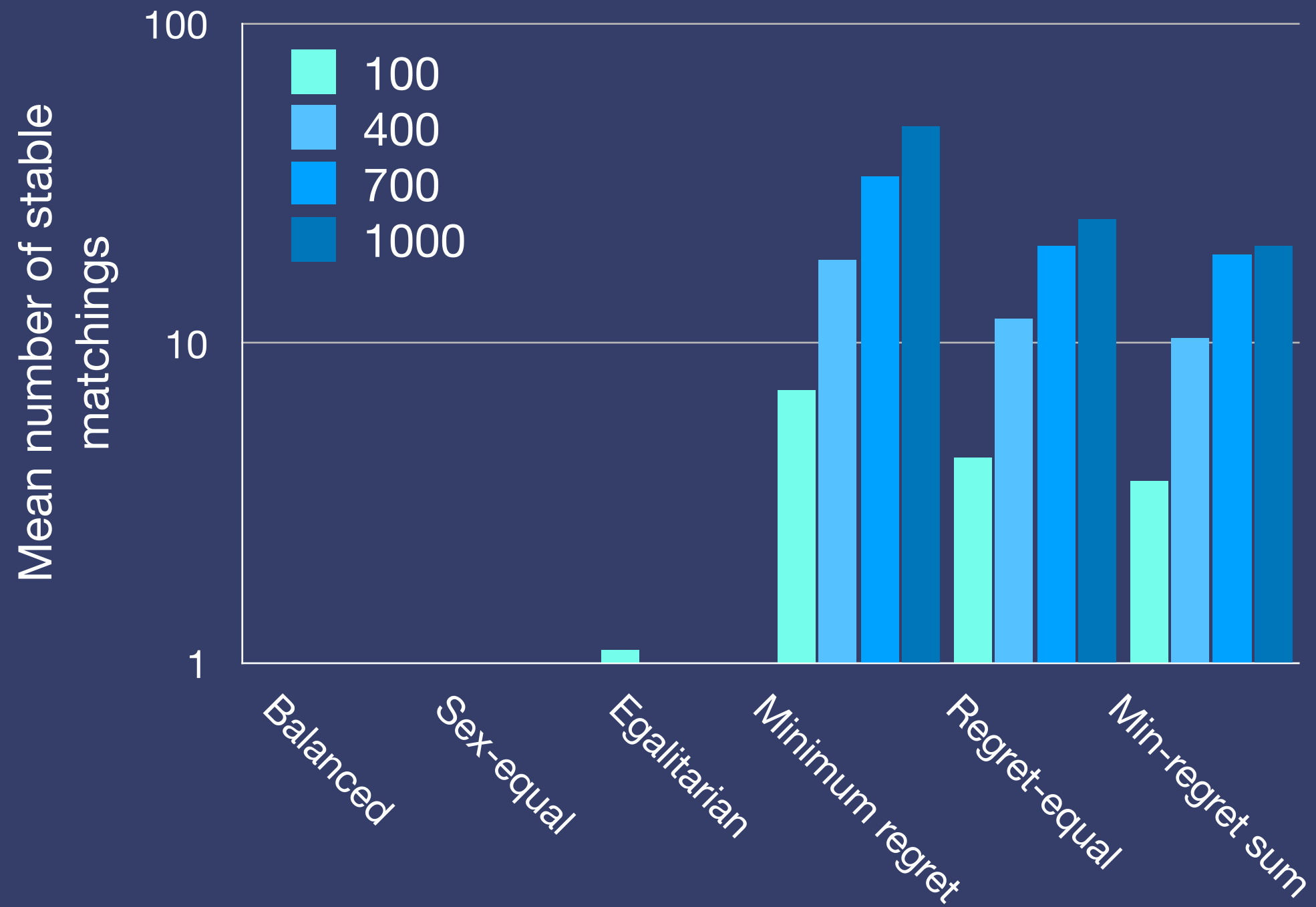
Sex-equal score for different optimal matchings

Sex-equal score for different optimal matchings



Frequency of different optimal stable matchings

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 - Open problem for ‘sex-equality’ $\rightarrow$ grouped-women-equality

Thank you

Summary

- Matching problems
- Fairness
- Finding fair stable matchings
- Experiments
- Future work: finding improved algorithms



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Research Council
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