Regret-equality in Stable Marriage

Frances Cooper
Joint work with: Prof David Manlove

Outline

Outline

- Matching problems

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- Fairness

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- Fairness
- Finding fair stable matchings

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- Experiments

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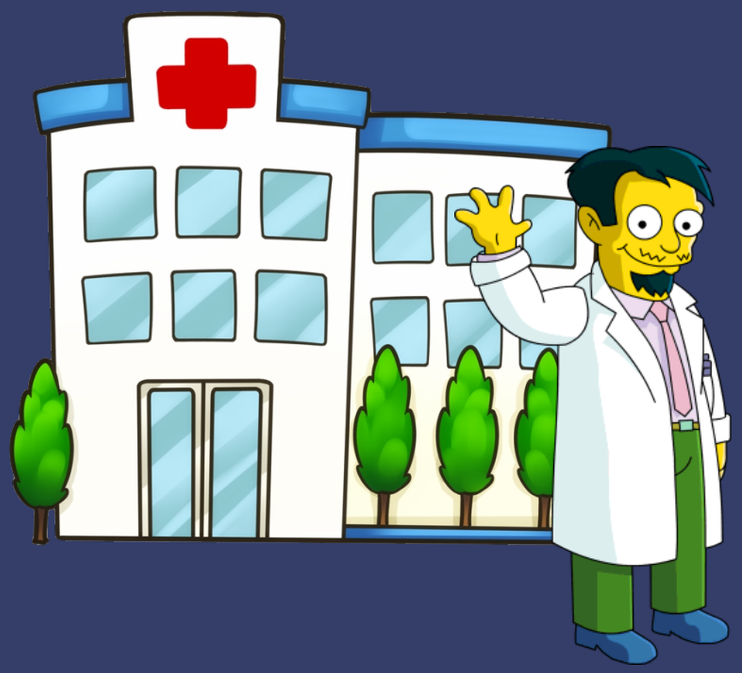
- Matching problems
- Fairness
- Finding fair stable matchings
- Experiments
- Future work

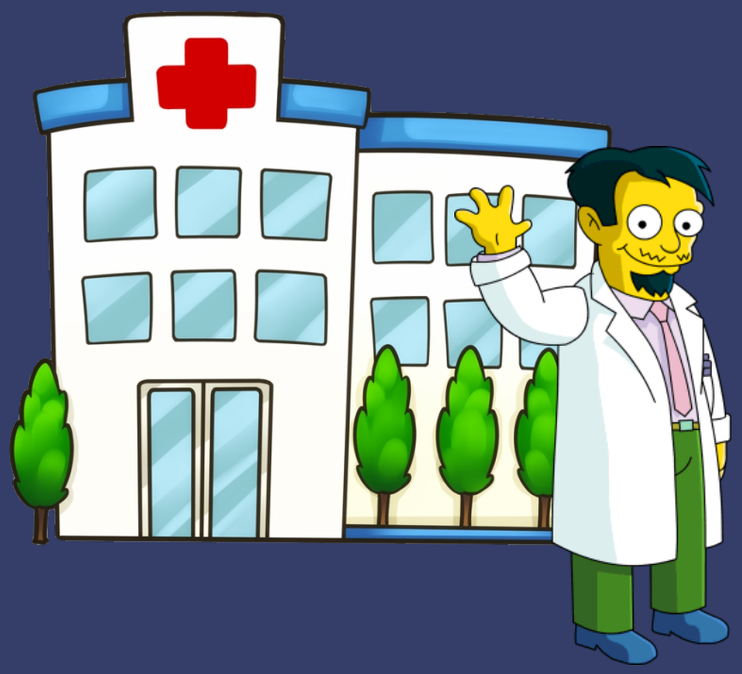
Matching Problems

Matching Problems

Matching Problems

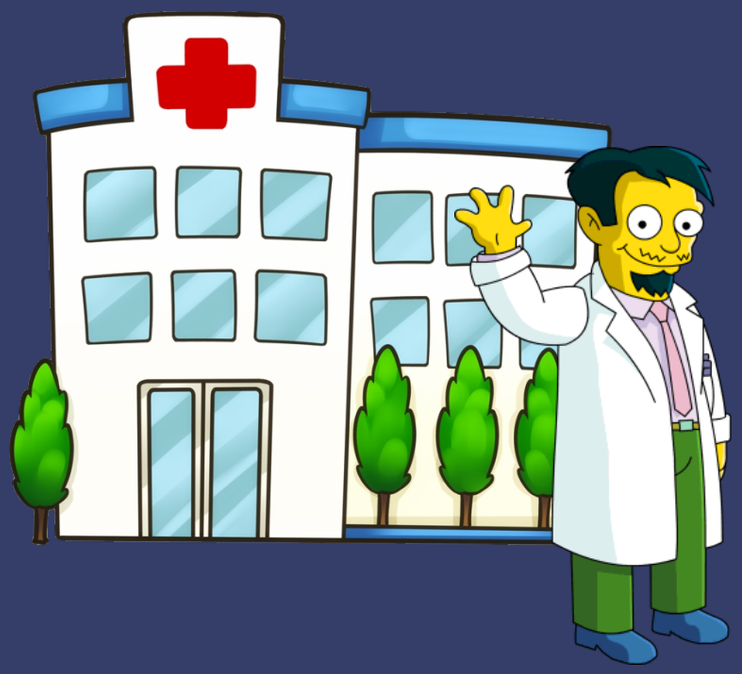


Matching Problems



- Assign one set of entities to another set of entities

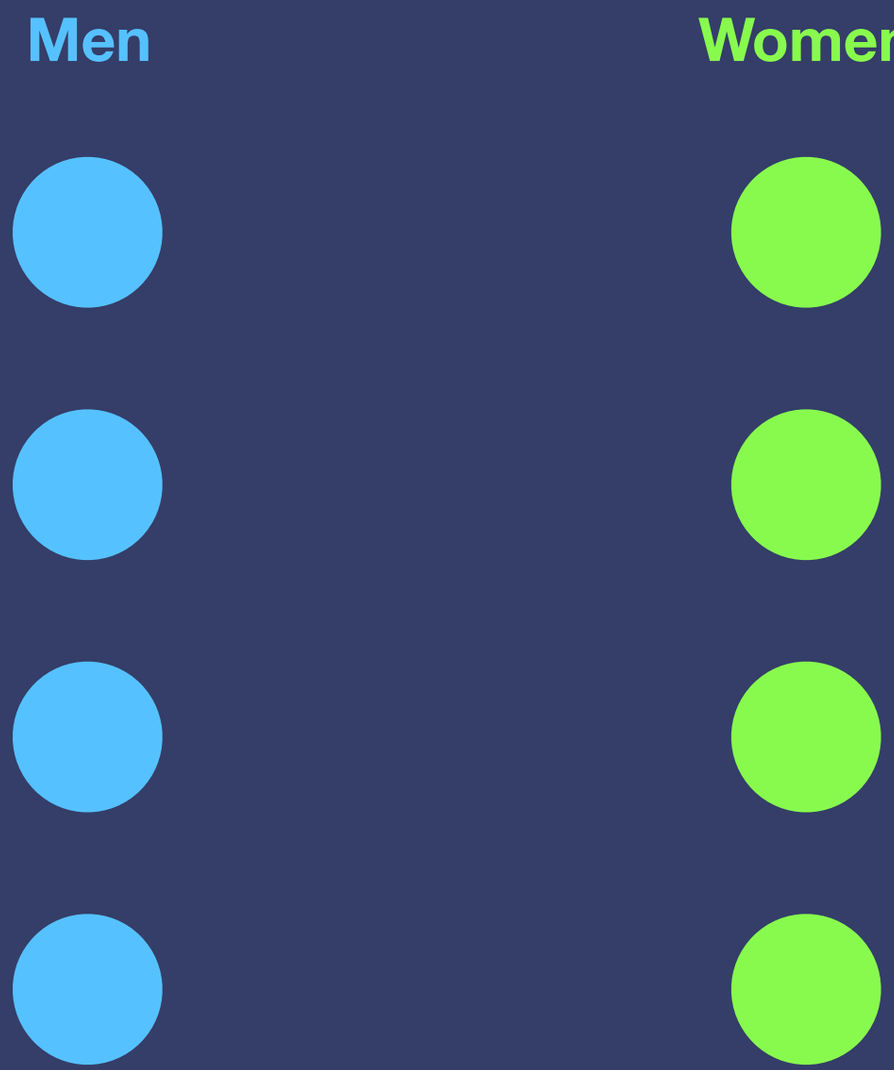
Matching Problems



- Assign one set of entities to another set of entities
- Based on preferences and capacities

Stable Marriage

Stable Marriage



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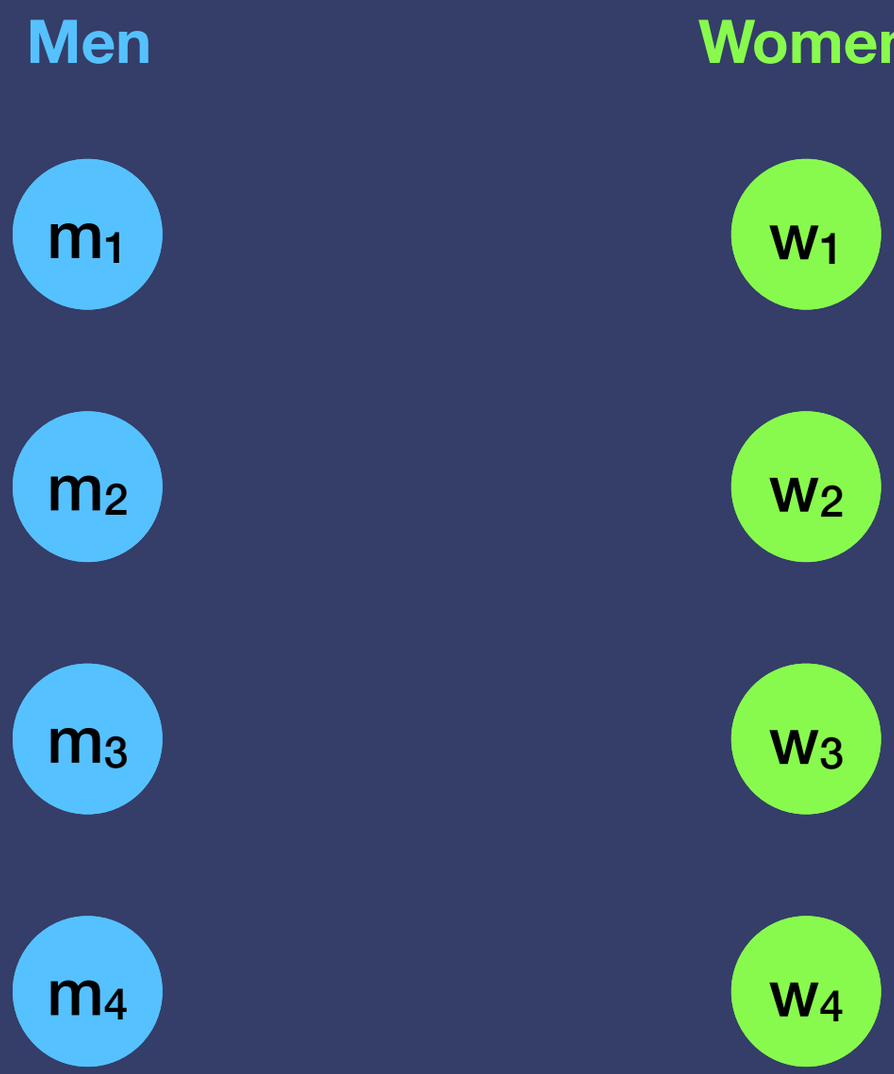

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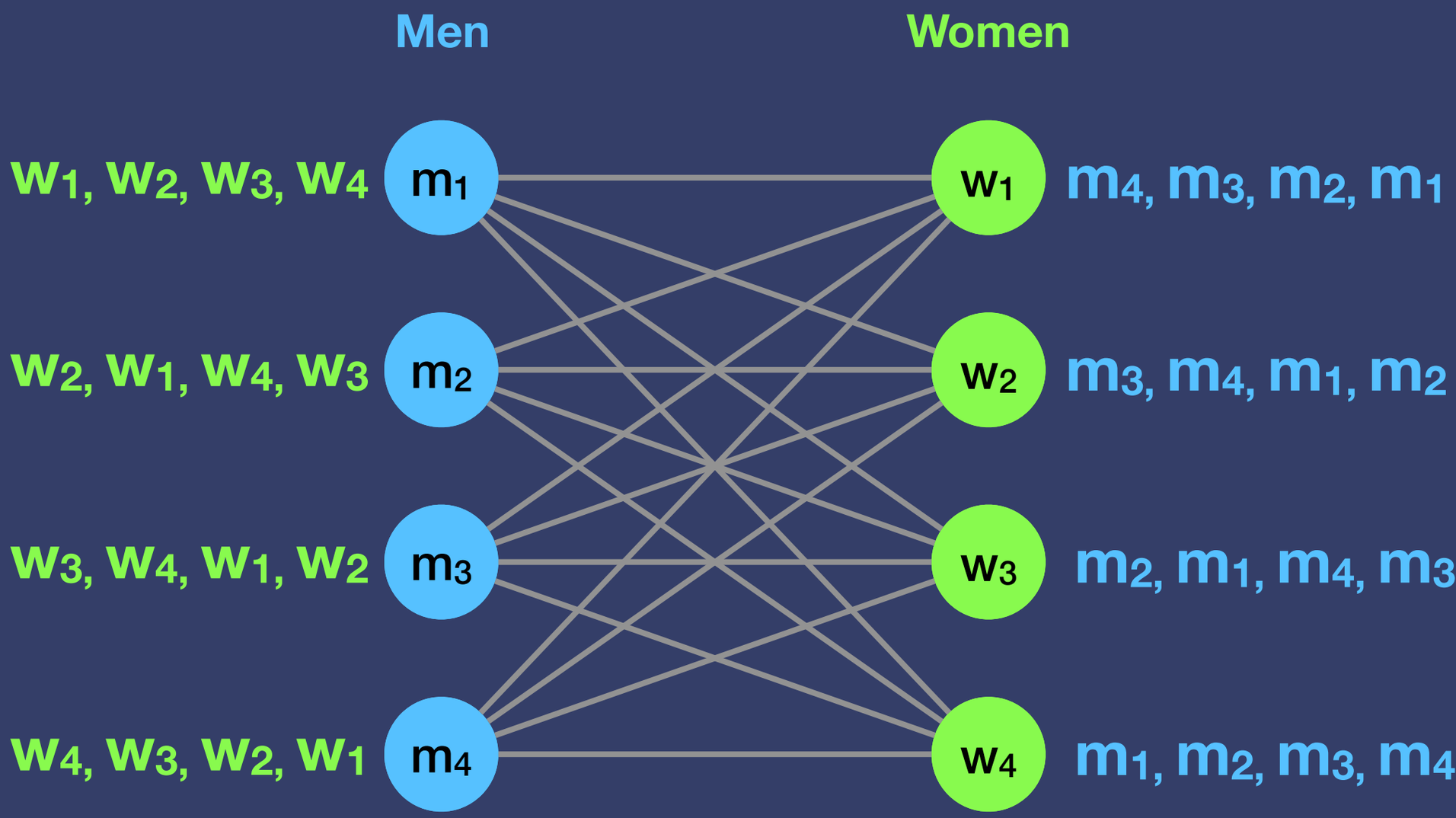

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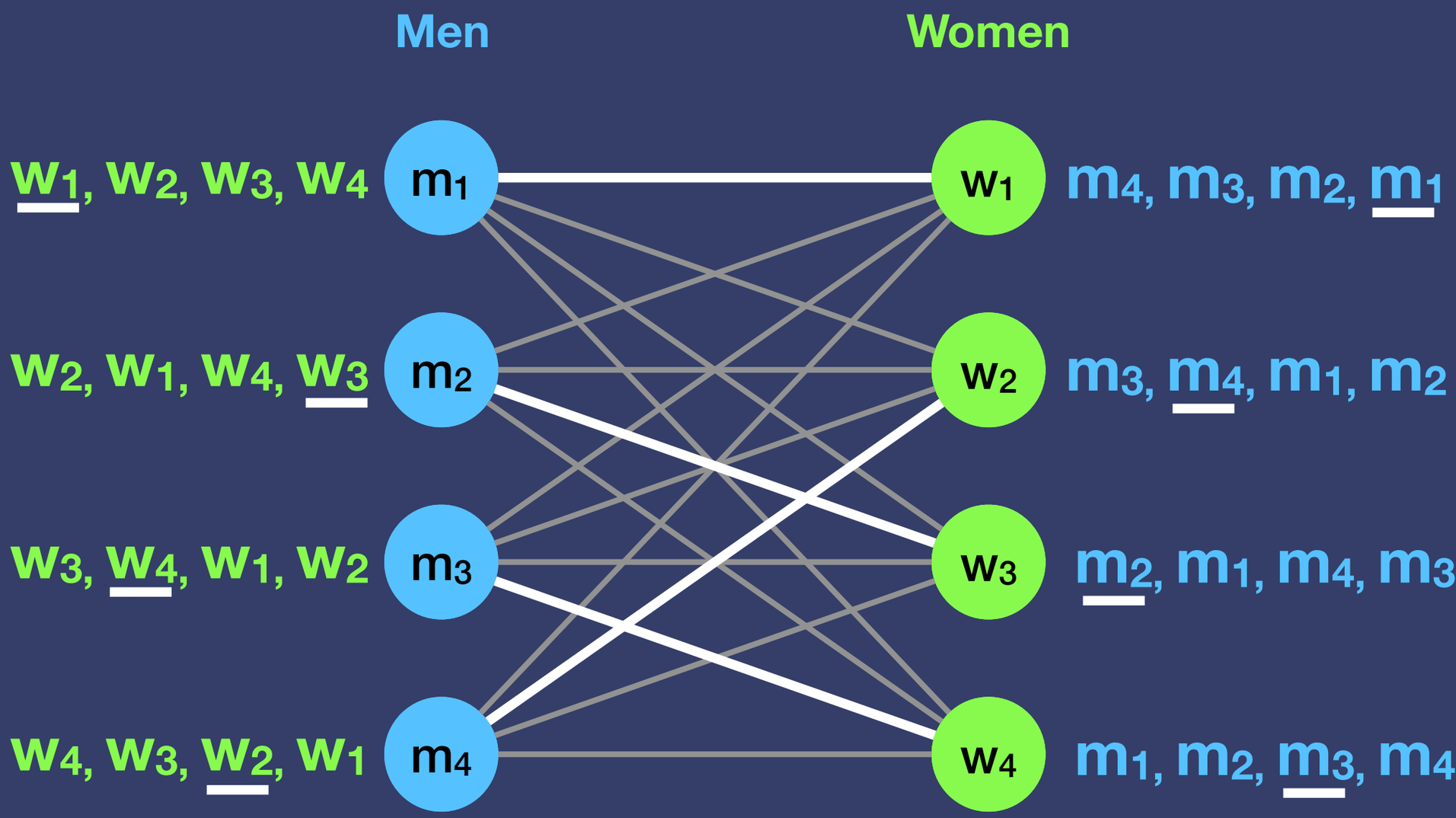
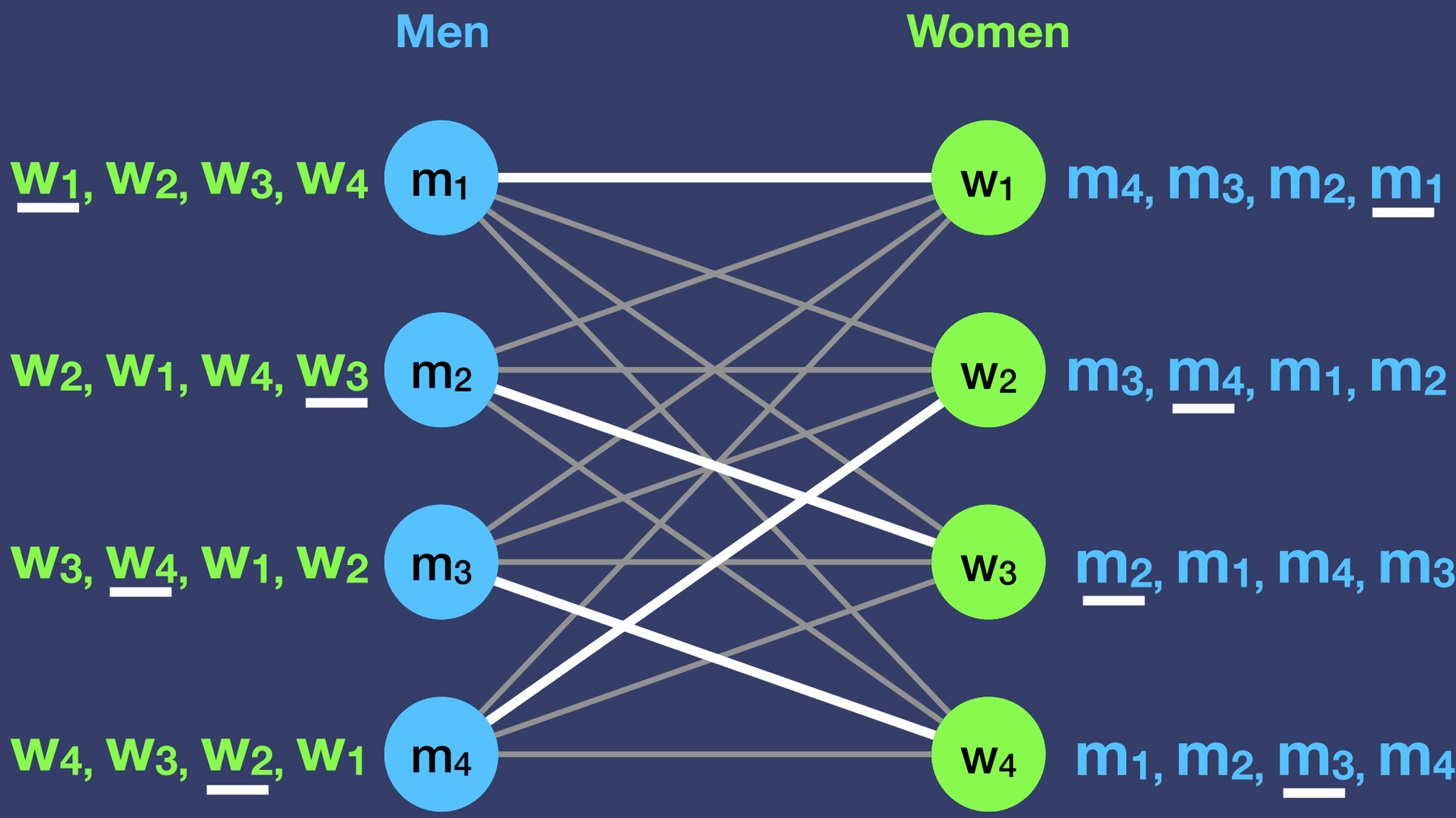


Rank

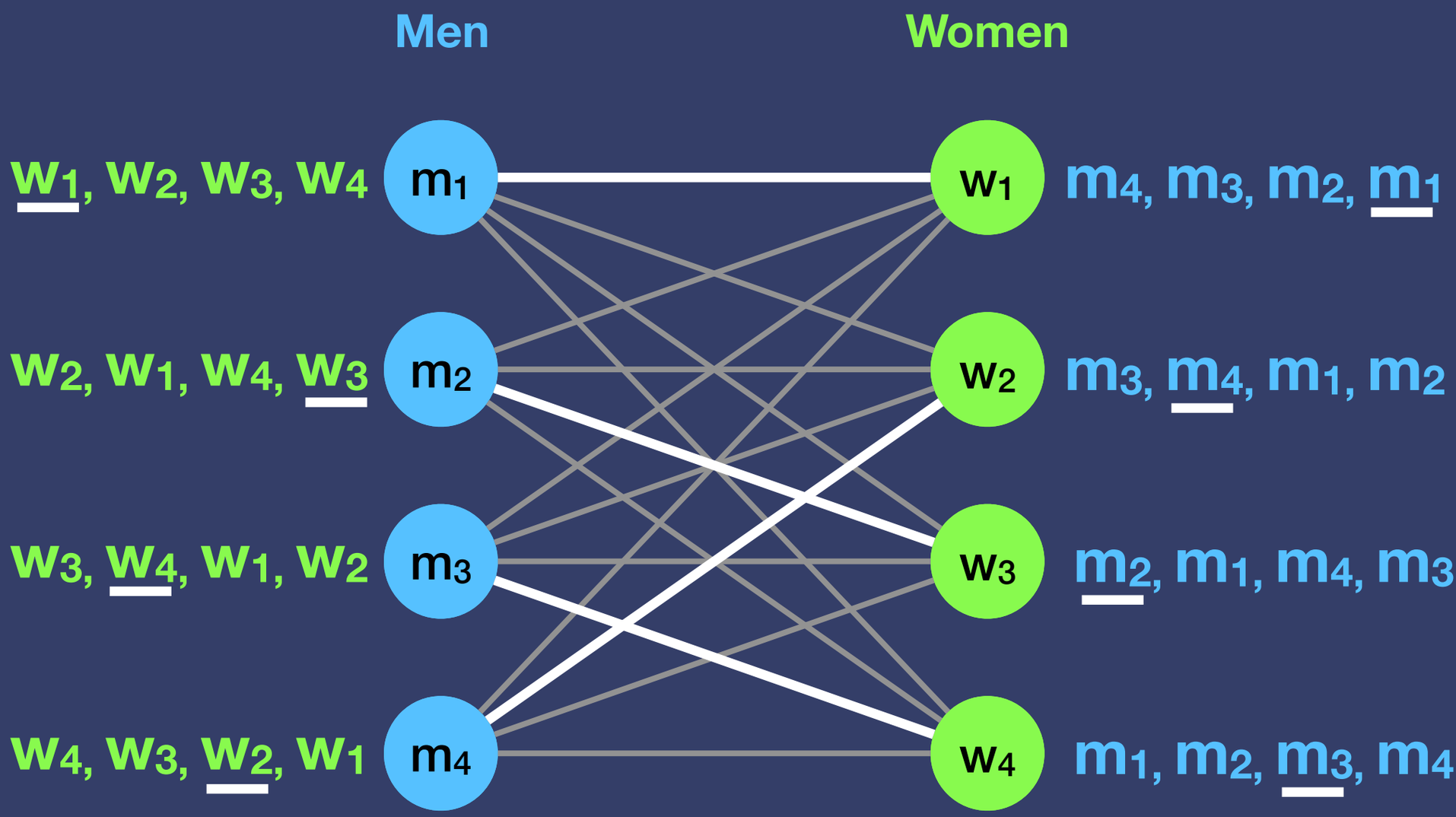
Stable Marriage



Rank

Stable Marriage

Cost: $c_U(M) = 10$, $c_W(M) = 10$

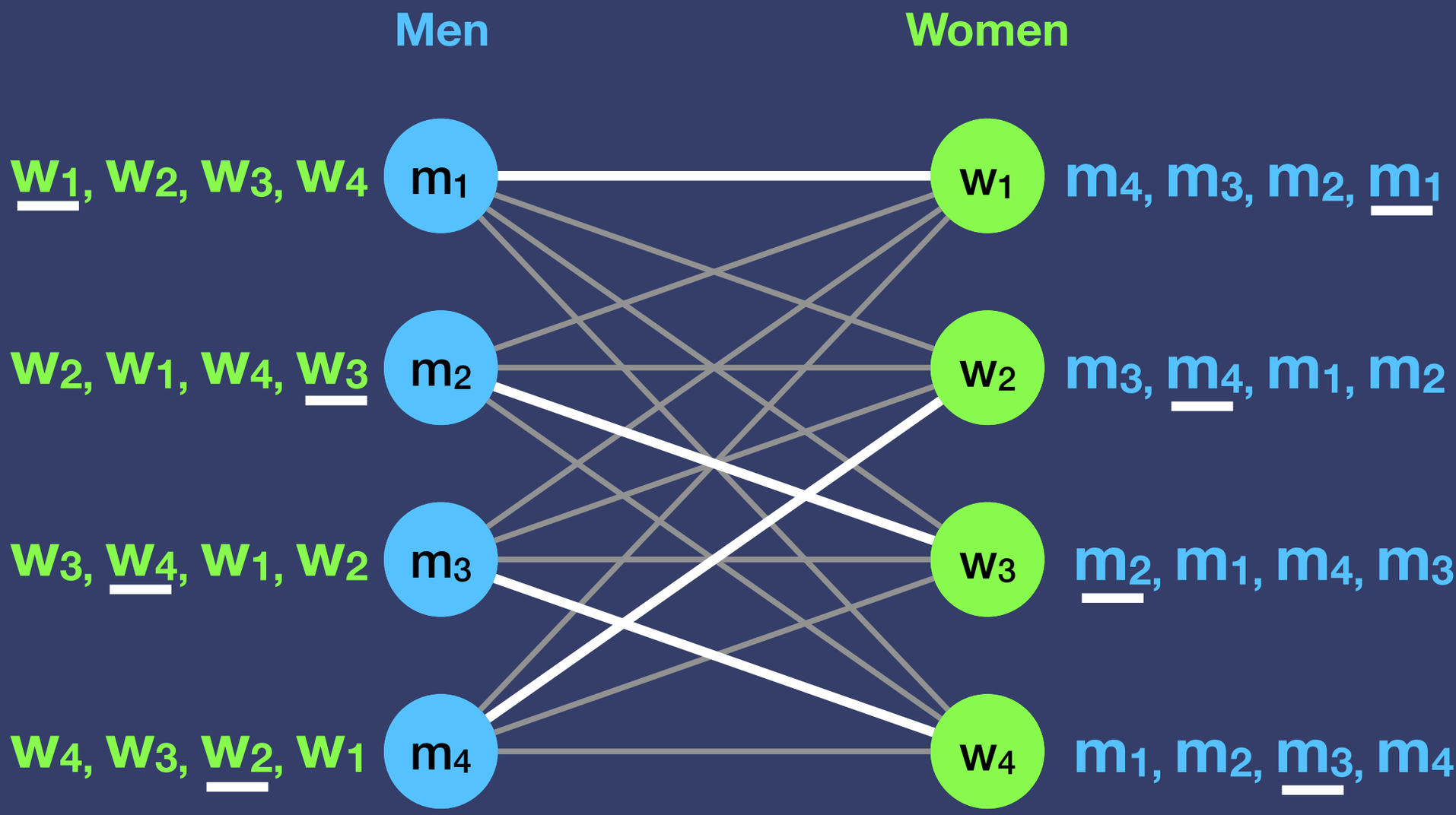


Stable Marriage

Rank

Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$



Stable Marriage

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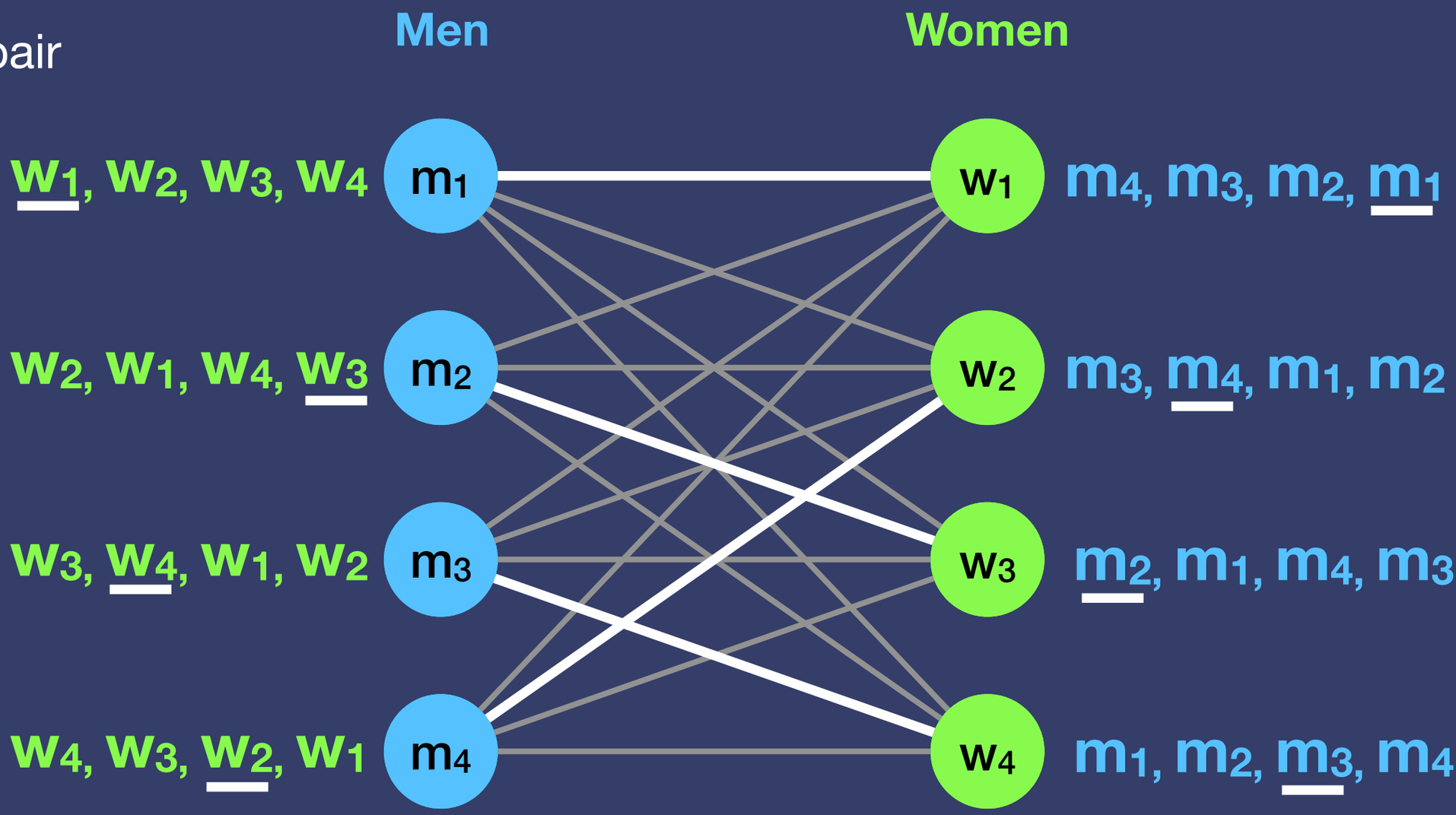
Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$

Blocking pair



Stable Marriage

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Degree: $d_U(M) = 4$, $d_W(M) = 4$

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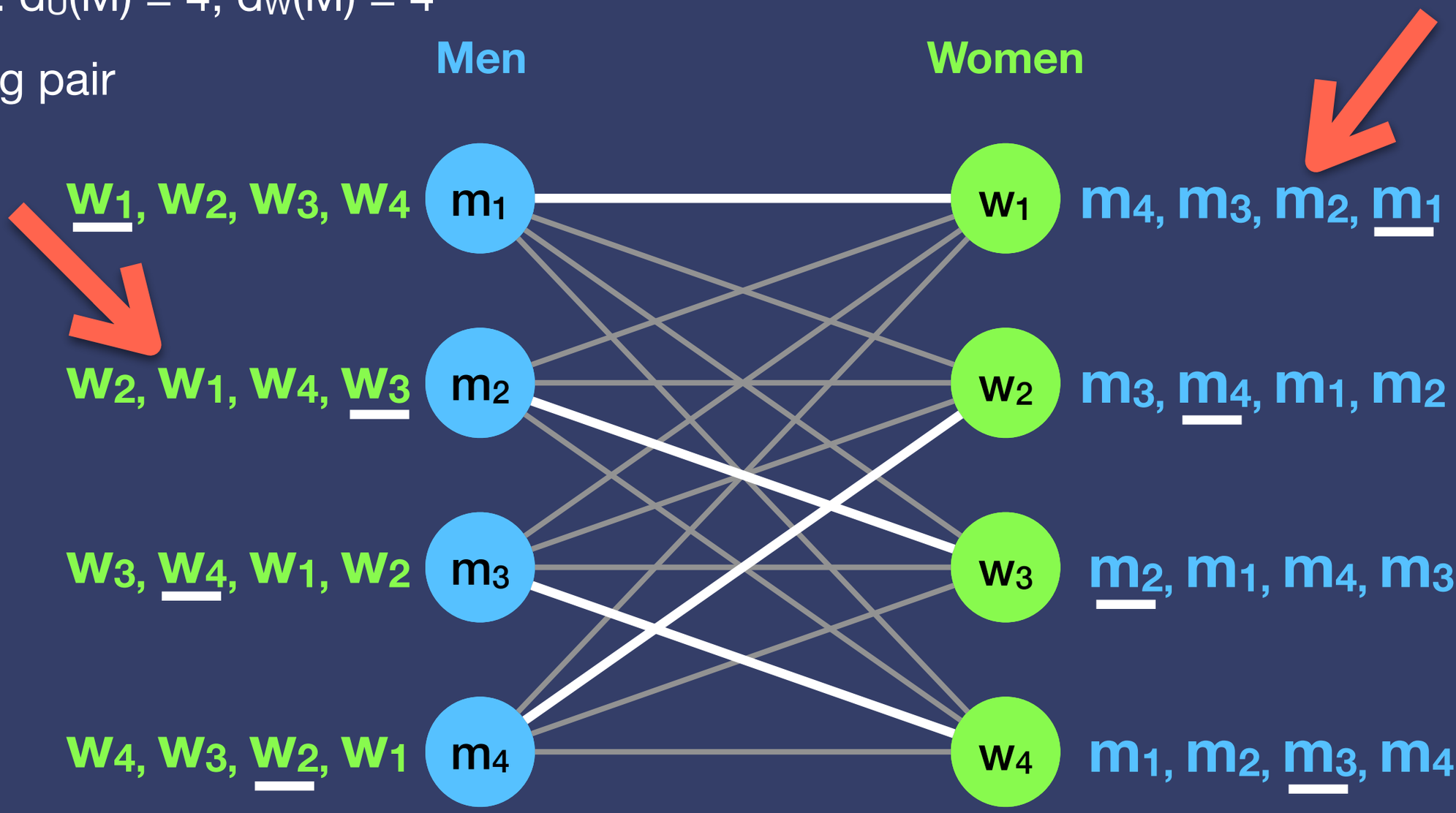

Stable Marriage

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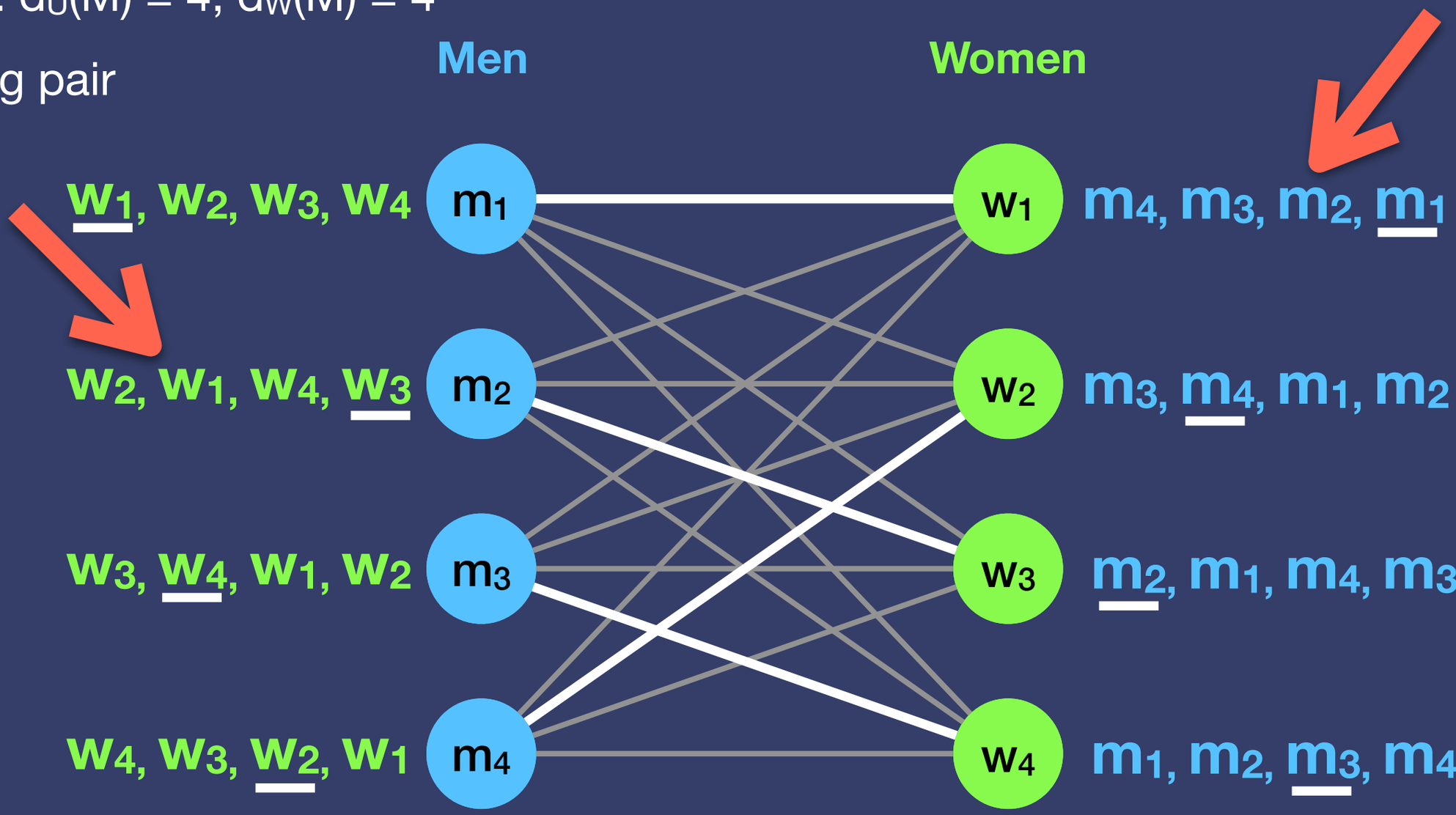

A **stable matching** is a matching with no blocking pairs

Stable Marriage

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- A **stable matching** is a matching with no blocking pairs
- Many stable matchings per instance

Stable Marriage

- A **stable matching** is a matching with no blocking pairs
- Many stable matchings per instance
- We can find a stable matching in linear time using the man-oriented or woman-oriented Gale-Shapley Algorithm. O(m) time where m is total length of preference lists

Stable Marriage

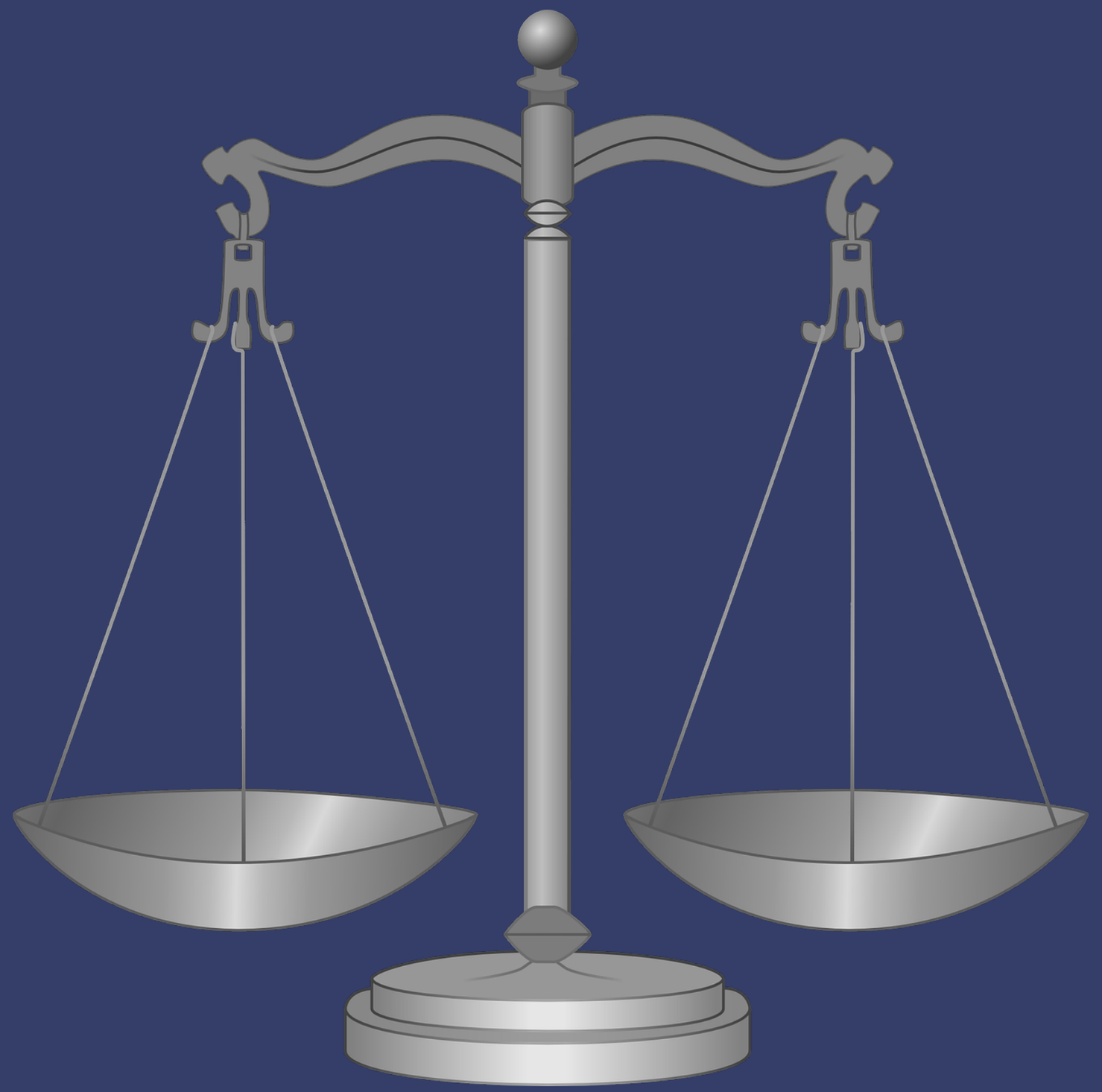
- A **stable matching** is a matching with no blocking pairs

- Many stable matchings per instance

- We can find a stable matching in linear time using the man-oriented or woman-oriented Gale-Shapley Algorithm. O(m) time where m is total length of preference lists

- Man-oriented Gale-Shapley Algorithm: finds a man-optimal (woman-pessimal) stable matching (and vice versa)

Fairness



Fairness

- Want to find a stable matching that provides some kind of equality between men and women

Fairness

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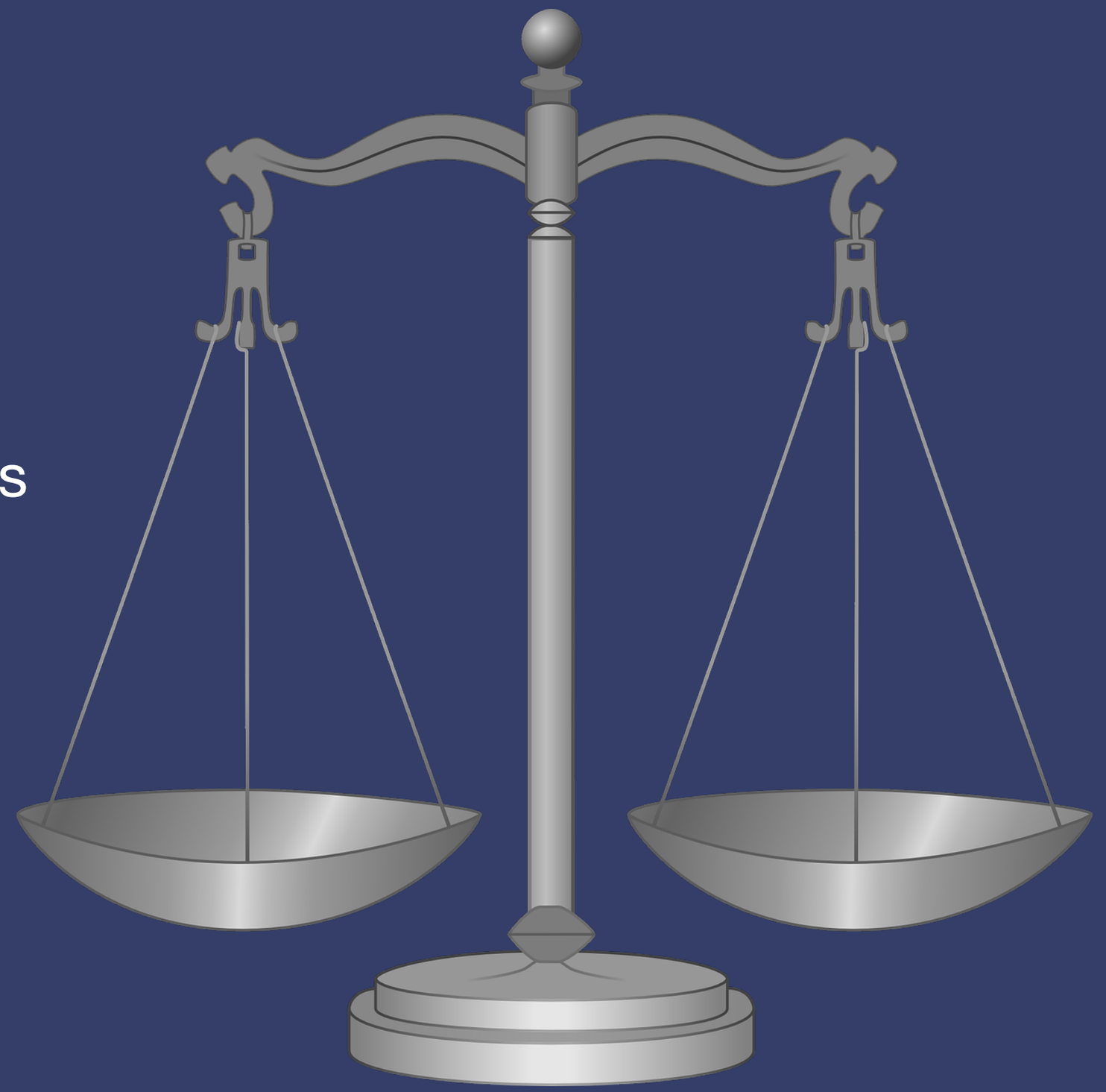
- Several different fairness measures



Fairness measures

Fairness measures

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Fairness measures

Among all stable matchings, find the stable matching that…

	Cost: $c_U(M)$, $c_W(M)$	Degree: $d_U(M)$, $d_W(M)$
Minimises the maximum		
Minimises the difference		
Minimises the sum		

Fairness measures

Among all stable matchings, find the stable matching that…

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum	**balanced score** Balanced stable matching NP-hard	
Minimises the difference		
Minimises the sum		

Fairness measures

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Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum	**balanced score** Balanced stable matching NP-hard	
Minimises the difference	**sex-equal score** Sex-equal stable matching NP-hard	
Minimises the sum		

Fairness measures

Among all stable matchings, find the stable matching that…

Cost: $c_U(M)$, $c_W(M)$ Degree: $d_U(M)$, $d_W(M)$

Minimises the maximum	**balanced score** Balanced stable matching NP-hard	
Minimises the difference	**sex-equal score** Sex-equal stable matching NP-hard	
Minimises the sum	**egalitarian cost** Egalitarian stable matching Poly	

Fairness measures

Among all stable matchings, find the stable matching that…

	Cost: $c_U(M)$, $c_W(M)$	Degree: $d_U(M)$, $d_W(M)$
Minimises the maximum	**balanced score** Balanced stable matching NP-hard	**degree** Minimum-regret stable matching Poly
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Fairness measures

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Minimises the maximum	**balanced score** Balanced stable matching NP-hard	**degree** Minimum-regret stable matching Poly
Minimises the difference	**sex-equal score** Sex-equal stable matching NP-hard	**regret-equal score** * Regret-equal stable matching ?
Minimises the sum	**egalitarian cost** Egalitarian stable matching Poly	

Fairness measures

Among all stable matchings, find the stable matching that…

	Cost: $c_U(M)$, $c_W(M)$	Degree: $d_U(M)$, $d_W(M)$
Minimises the maximum	**balanced score** Balanced stable matching NP-hard	**degree** Minimum-regret stable matching Poly
Minimises the difference	**sex-equal score** Sex-equal stable matching NP-hard	**regret-equal score** * Regret-equal stable matching ?
Minimises the sum	**egalitarian cost** Egalitarian stable matching Poly	**regret sum score** * Min-regret sum stable matching ?

Fairness measures (degree based)

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10 stable matchings for this instance

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10 stable matchings for this instance

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, $\underline{w_1}$, w_4, w_3
m_3: w_3, $\underline{w_4}$, w_1, w_2
m_4: w_4, w_3, $\underline{w_2}$, w_1

w_1: m_4, m_3, $\underline{m_2}$, m_1
w_2: m_3, $\underline{m_4}$, m_2, m_1
w_3: m_2, $\underline{m_1}$, m_4, m_3
w_4: m_1, m_2, $\underline{m_3}$, m_4

Fairness measures (degree based)

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m_1: w_1, w_2, $\underline{w_3}$, w_4 — w_1: m_4, m_3, $\underline{m_2}$, m_1
m_2: w_2, $\underline{w_1}$, w_4, w_3 — w_2: m_3, $\underline{m_4}$, m_2, m_1
m_3: w_3, $\underline{w_4}$, w_1, w_2 — w_3: m_2, $\underline{m_1}$, m_4, m_3
m_4: w_4, w_3, $\underline{w_2}$, w_1 — w_4: m_1, m_2, $\underline{m_3}$, m_4

m_1: w_1, w_2, $\underline{w_3}$, w_4 — w_1: m_4, $\underline{m_3}$, m_2, m_1
m_2: w_2, w_1, $\underline{w_4}$, w_3 — w_2: m_3, $\underline{m_4}$, m_2, m_1
m_3: w_3, w_4, $\underline{w_1}$, w_2 — w_3: m_2, $\underline{m_1}$, m_4, m_3
m_4: w_4, w_3, $\underline{w_2}$, w_1 — w_4: m_1, $\underline{m_2}$, m_3, m_4

Fairness measures (degree based)

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m_1: w_1, w_2, $\underline{w_3}$, w_4	w_1: m_4, m_3, $\underline{m_2}$, m_1
m_2: w_2, $\underline{w_1}$, w_4, w_3	w_2: m_3, $\underline{m_4}$, m_2, m_1
m_3: w_3, $\underline{w_4}$, w_1, w_2	w_3: m_2, $\underline{m_1}$, m_4, m_3
m_4: w_4, w_3, $\underline{w_2}$, w_1	w_4: m_1, m_2, $\underline{m_3}$, m_4
m_1: w_1, w_2, $\underline{w_3}$, w_4	w_1: m_4, $\underline{m_3}$, m_2, m_1
m_2: w_2, w_1, $\underline{w_4}$, w_3	w_2: m_3, $\underline{m_4}$, m_2, m_1
m_3: w_3, w_4, $\underline{w_1}$, w_2	w_3: m_2, $\underline{m_1}$, m_4, m_3
m_4: w_4, w_3, $\underline{w_2}$, w_1	w_4: m_1, $\underline{m_2}$, m_3, m_4
m_1: $\underline{w_1}$, w_2, w_3, w_4	w_1: m_4, m_3, m_2, $\underline{m_1}$
m_2: $\underline{w_2}$, w_1, w_4, w_3	w_2: m_3, m_4, $\underline{m_2}$, m_1
m_3: $\underline{w_3}$, w_4, w_1, w_2	w_3: m_2, m_1, m_4, $\underline{m_3}$
m_4: $\underline{w_4}$, w_3, w_2, w_1	w_4: m_1, m_2, m_3, $\underline{m_4}$

Fairness measures (degree based)

10 stable matchings for this instance

m_1: w_1, w_2, $\underline{w_3}$, w_4 — w_1: m_4, m_3, $\underline{m_2}$, m_1
m_2: w_2, $\underline{w_1}$, w_4, w_3 — w_2: m_3, $\underline{m_4}$, m_2, m_1
m_3: w_3, $\underline{w_4}$, w_1, w_2 — w_3: m_2, $\underline{m_1}$, m_4, m_3
m_4: w_4, w_3, $\underline{w_2}$, w_1 — w_4: m_1, m_2, $\underline{m_3}$, m_4

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m_2: w_2, w_1, $\underline{w_4}$, w_3 — w_2: m_3, $\underline{m_4}$, m_2, m_1
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m_3: $\underline{w_3}$, w_4, w_1, w_2 — w_3: m_2, m_1, m_4, $\underline{m_3}$
m_4: $\underline{w_4}$, w_3, w_2, w_1 — w_4: m_1, m_2, m_3, $\underline{m_4}$

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

Fairness measures (degree based)

10 stable matchings for this instance

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, $\underline{w_1}$, w_4, w_3
m_3: w_3, $\underline{w_4}$, w_1, w_2
m_4: w_4, w_3, $\underline{w_2}$, w_1

w_1: m_4, m_3, $\underline{m_2}$, m_1
w_2: m_3, $\underline{m_4}$, m_2, m_1
w_3: m_2, $\underline{m_1}$, m_4, m_3
w_4: m_1, m_2, $\underline{m_3}$, m_4

Degree: 3
Regret-equality score: 0
Min-regret sum score: 6

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, w_1, $\underline{w_4}$, w_3
m_3: w_3, w_4, $\underline{w_1}$, w_2
m_4: w_4, w_3, $\underline{w_2}$, w_1

w_1: m_4, $\underline{m_3}$, m_2, m_1
w_2: m_3, $\underline{m_4}$, m_2, m_1
w_3: m_2, $\underline{m_1}$, m_4, m_3
w_4: m_1, $\underline{m_2}$, m_3, m_4

Degree: 3
Regret-equality score: 1
Min-regret sum score: 5

m_1: $\underline{w_1}$, w_2, w_3, w_4
m_2: $\underline{w_2}$, w_1, w_4, w_3
m_3: $\underline{w_3}$, w_4, w_1, w_2
m_4: $\underline{w_4}$, w_3, w_2, w_1

w_1: m_4, m_3, m_2, $\underline{m_1}$
w_2: m_3, m_4, $\underline{m_2}$, m_1
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Fairness measures (degree based)

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m_1: w_1, w_2, $\underline{w_3}$, w_4 m_2: w_2, $\underline{w_1}$, w_4, w_3 m_3: w_3, $\underline{w_4}$, w_1, w_2 m_4: w_4, w_3, $\underline{w_2}$, w_1	w_1: m_4, m_3, $\underline{m_2}$, m_1 w_2: m_3, $\underline{m_4}$, m_2, m_1 w_3: m_2, $\underline{m_1}$, m_4, m_3 w_4: m_1, m_2, $\underline{m_3}$, m_4	Degree: 3 Regret-equality score: 0 Min-regret sum score: 6
m_1: w_1, w_2, $\underline{w_3}$, w_4 m_2: w_2, w_1, $\underline{w_4}$, w_3 m_3: w_3, w_4, $\underline{w_1}$, w_2 m_4: w_4, w_3, $\underline{w_2}$, w_1	w_1: m_4, $\underline{m_3}$, m_2, m_1 w_2: m_3, $\underline{m_4}$, m_2, m_1 w_3: m_2, $\underline{m_1}$, m_4, m_3 w_4: m_1, $\underline{m_2}$, m_3, m_4	Degree: 3 Regret-equality score: 1 Min-regret sum score: 5
m_1: $\underline{w_1}$, w_2, w_3, w_4 m_2: $\underline{w_2}$, w_1, w_4, w_3 m_3: $\underline{w_3}$, w_4, w_1, w_2 m_4: $\underline{w_4}$, w_3, w_2, w_1	w_1: m_4, m_3, m_2, $\underline{m_1}$ w_2: m_3, m_4, $\underline{m_2}$, m_1 w_3: m_2, m_1, m_4, $\underline{m_3}$ w_4: m_1, m_2, m_3, $\underline{m_4}$	Degree: 4 Regret-equality score: 3 Min-regret sum score: 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1: w_1, w_2, $\underline{w_3}$, w_4 m_2: w_2, $\underline{w_1}$, w_4, w_3 m_3: w_3, $\underline{w_4}$, w_1, w_2 m_4: w_4, w_3, $\underline{w_2}$, w_1	w_1: m_4, m_3, $\underline{m_2}$, m_1 w_2: m_3, $\underline{m_4}$, m_2, m_1 w_3: m_2, $\underline{m_1}$, m_4, m_3 w_4: m_1, m_2, $\underline{m_3}$, m_4	Degree: 3 Regret-equality score: 0 Min-regret sum score: 6
m_1: w_1, w_2, $\underline{w_3}$, w_4 m_2: w_2, w_1, $\underline{w_4}$, w_3 m_3: w_3, w_4, $\underline{w_1}$, w_2 m_4: w_4, w_3, $\underline{w_2}$, w_1	w_1: m_4, $\underline{m_3}$, m_2, m_1 w_2: m_3, $\underline{m_4}$, m_2, m_1 w_3: m_2, $\underline{m_1}$, m_4, m_3 w_4: m_1, $\underline{m_2}$, m_3, m_4	Degree: 3 Regret-equality score: 1 Min-regret sum score: 5
m_1: $\underline{w_1}$, w_2, w_3, w_4 m_2: $\underline{w_2}$, w_1, w_4, w_3 m_3: $\underline{w_3}$, w_4, w_1, w_2 m_4: $\underline{w_4}$, w_3, w_2, w_1	w_1: m_4, m_3, m_2, $\underline{m_1}$ w_2: m_3, m_4, $\underline{m_2}$, m_1 w_3: m_2, m_1, m_4, $\underline{m_3}$ w_4: m_1, m_2, m_3, $\underline{m_4}$	Degree: 4 Regret-equality score: 3 Min-regret sum score: 5

Over all stable matchings:
Minimum degree = 3
Minimum regret-equality score = 0
Minimum regret sum score = 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, $\underline{w_1}$, w_4, w_3
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m_4: w_4, w_3, $\underline{w_2}$, w_1

w_1: m_4, m_3, $\underline{m_2}$, m_1
w_2: m_3, $\underline{m_4}$, m_2, m_1
w_3: m_2, $\underline{m_1}$, m_4, m_3
w_4: m_1, m_2, $\underline{m_3}$, m_4

Min-regret & Regret-equal
Degree: 3
Regret-equality score: 0
Min-regret sum score: 6

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, w_1, $\underline{w_4}$, w_3
m_3: w_3, w_4, $\underline{w_1}$, w_2
m_4: w_4, w_3, $\underline{w_2}$, w_1

w_1: m_4, $\underline{m_3}$, m_2, m_1
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w_1: m_4, m_3, m_2, $\underline{m_1}$
w_2: m_3, m_4, $\underline{m_2}$, m_1
w_3: m_2, m_1, m_4, $\underline{m_3}$
w_4: m_1, m_2, m_3, $\underline{m_4}$

Degree: 4
Regret-equality score: 3
Min-regret sum score: 5

Over all stable matchings:
Minimum degree = 3
Minimum regret-equality score = 0
Minimum regret sum score = 5

Fairness measures (degree based)

10 stable matchings for this instance

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, $\underline{w_1}$, w_4, w_3
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w_3: m_2, $\underline{m_1}$, m_4, m_3
w_4: m_1, m_2, $\underline{m_3}$, m_4

Min-regret & Regret-equal

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

m_1: w_1, w_2, $\underline{w_3}$, w_4
m_2: w_2, w_1, $\underline{w_4}$, w_3
m_3: w_3, w_4, $\underline{w_1}$, w_2
m_4: w_4, w_3, $\underline{w_2}$, w_1

w_1: m_4, $\underline{m_3}$, m_2, m_1
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w_3: m_2, $\underline{m_1}$, m_4, m_3
w_4: m_1, $\underline{m_2}$, m_3, m_4

Min-regret & Min-regret sum

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

m_1: $\underline{w_1}$, w_2, w_3, w_4
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m_3: $\underline{w_3}$, w_4, w_1, w_2
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Min-regret sum score: 5

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Finding a Regret-Equal Stable Matching

Rotations

Rotations

- Rotation - series of man-woman pairs that take us from one stable matching to another when permuted

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R_1 m_1 m_4
w_2 w_3

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R_1

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R_1

M_2 m_1 m_2 m_3 m_4
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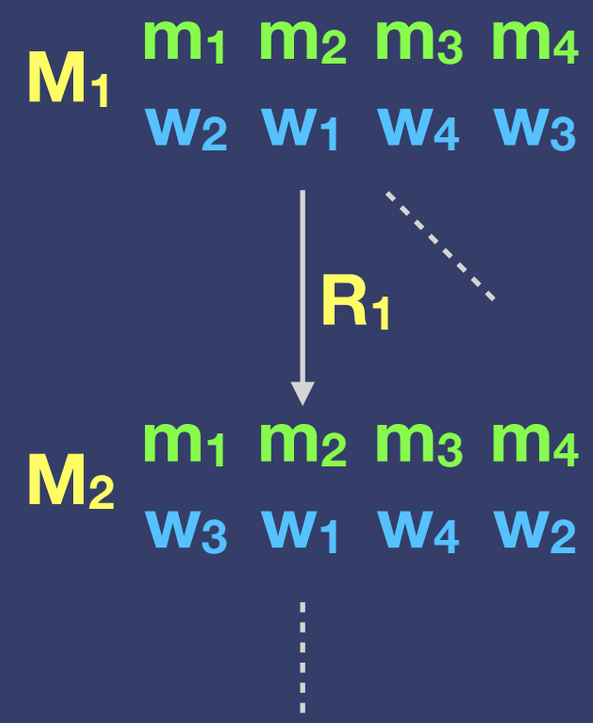
- Can only eliminate *exposed* rotations



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w_2 w_3

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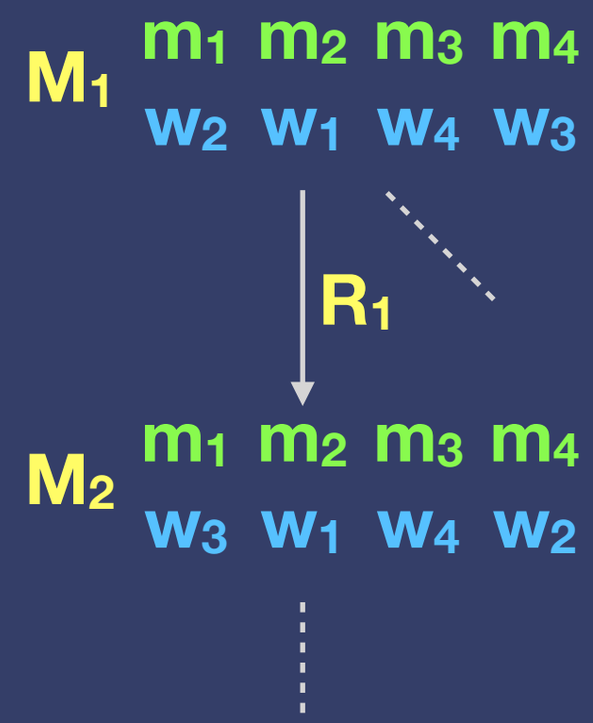

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- Rotations form a structure to allow enumeration of all stable matchings. All rotation makes some men worse off and some women better off

Algorithm

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1. Find the man-optimal stable matching M_0

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   - Each man has their best partner in any stable matching.
     Say **$d_U(M_0) = 2$** and **$d_W(M_0) = 5$** $d(M_0) = (2, 5)$

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   - Then, a regret equal stable matching must exist within the following degrees pairs:

r-e score: 3	(2, 5)
r-e score: 2	(2, 4) (3, 5)
r-e score: 1	(2, 3) (3, 4) (4, 5)
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why are these the only possible degrees?

- M_0 has a r-e score of 3
- men can only get worse
- women can only get better

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3. For each man m and for each column c:

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    1. rotate m down to c (if possible)

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3. For each man m and for each column c:

    1. rotate m down to c (if possible)

        2. rotate women down column c who have worst rank

r-e score: 3 (2, 5)
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- Stop iterating women up the column when **$d_U(M)$** >= **$d_W(M)$**
- Save the best matching as you go

Time complexity

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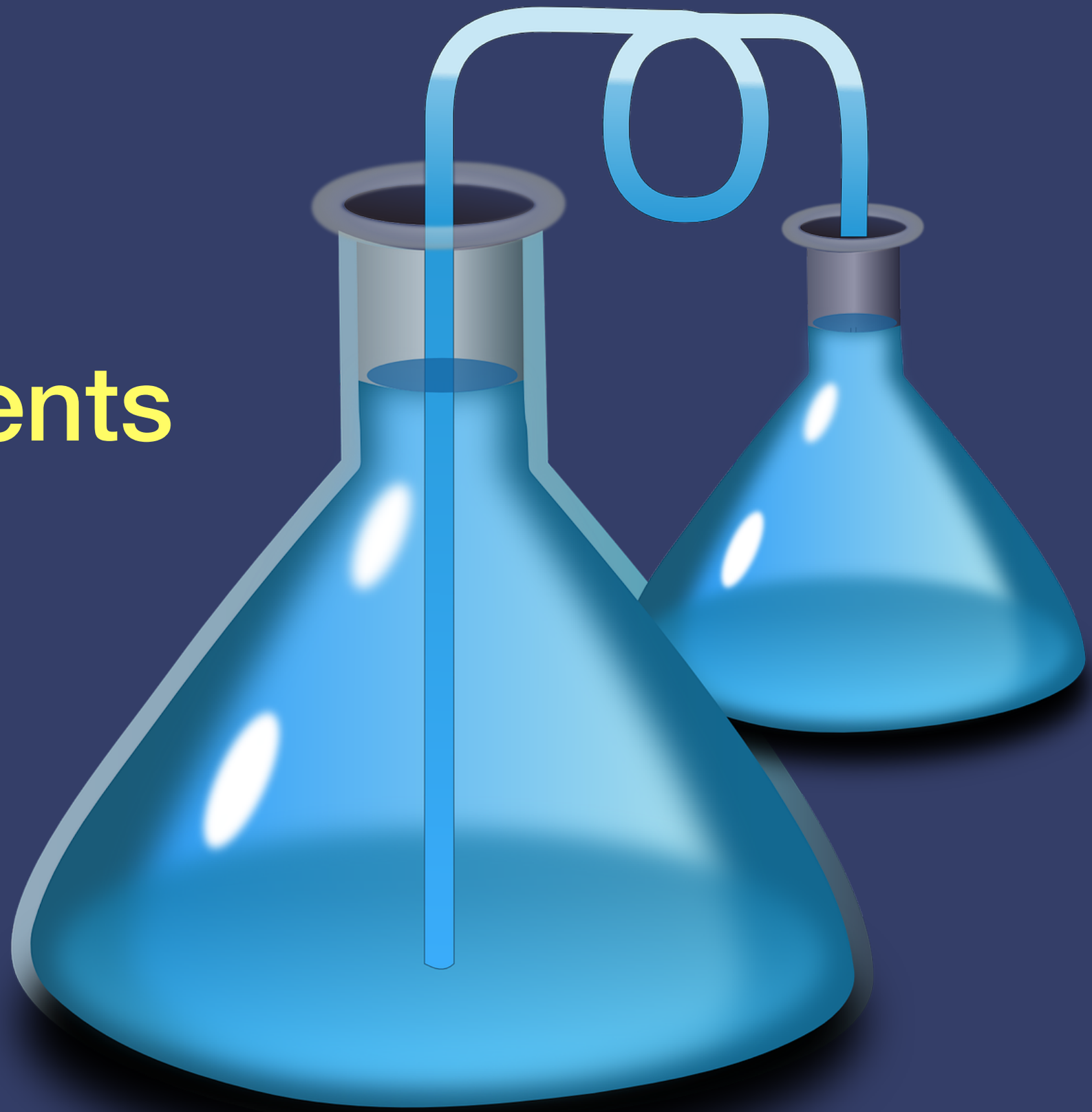
★ Total O(nmc) ★

Experiments



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    - CPLEX up to size n = 50 for the enumeration algorithm

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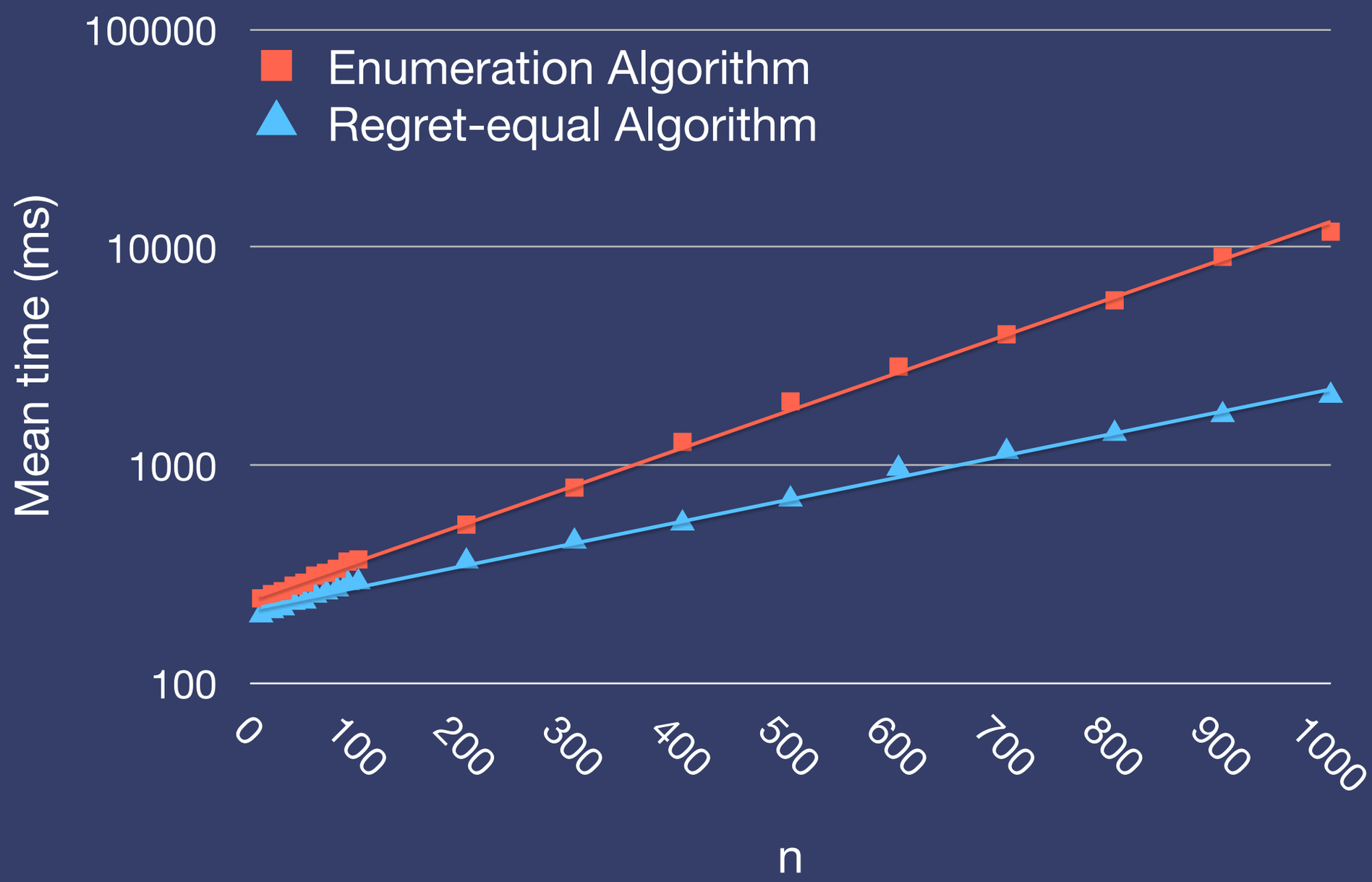
100000
10000
1000
100
Enumeration Algorithm
Regret-equal Algorithm
Mean time (ms)
0
100
200
300
400
500
600
700
800
900
1000
n

Regret-equality score for different optimal matchings

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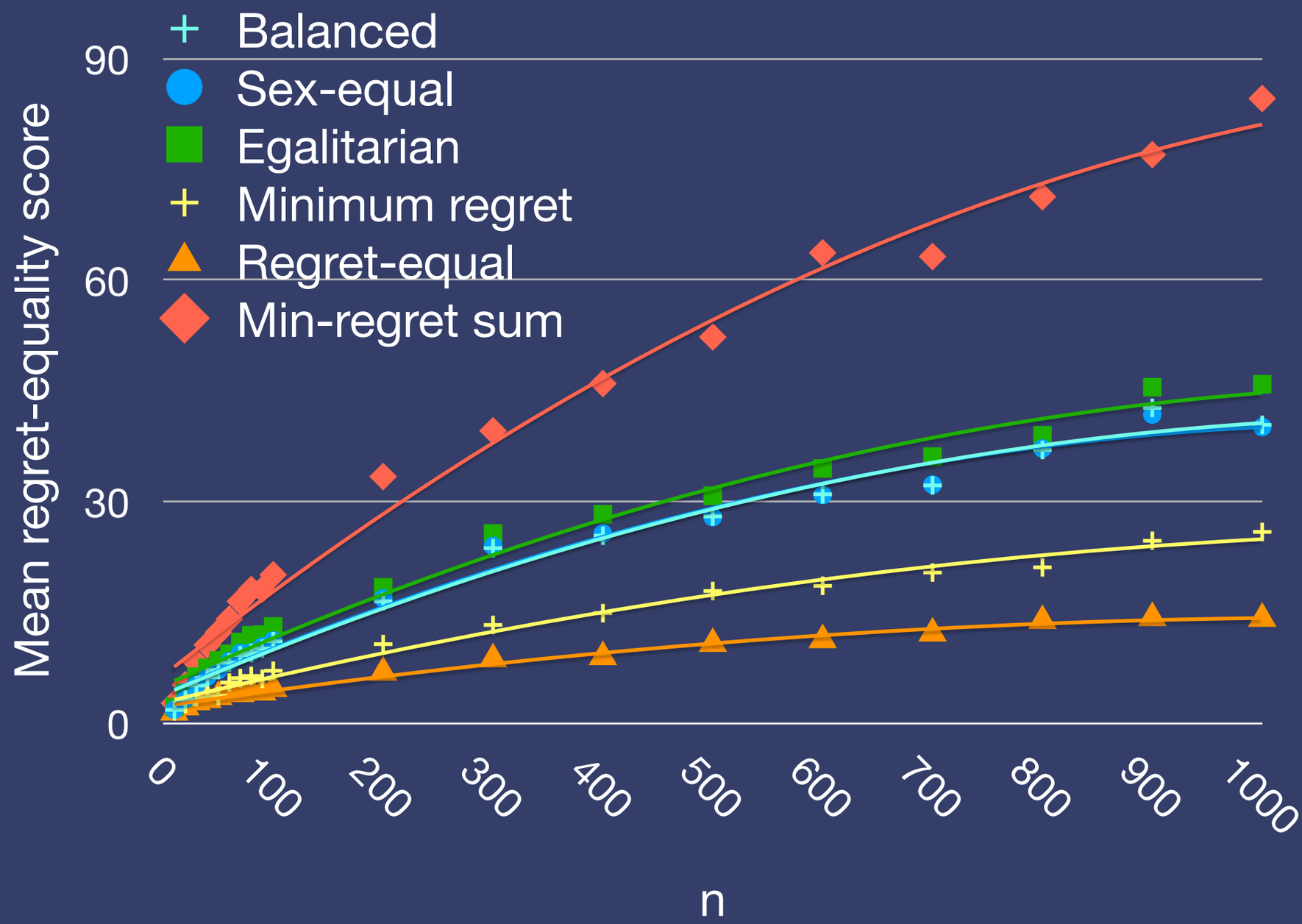
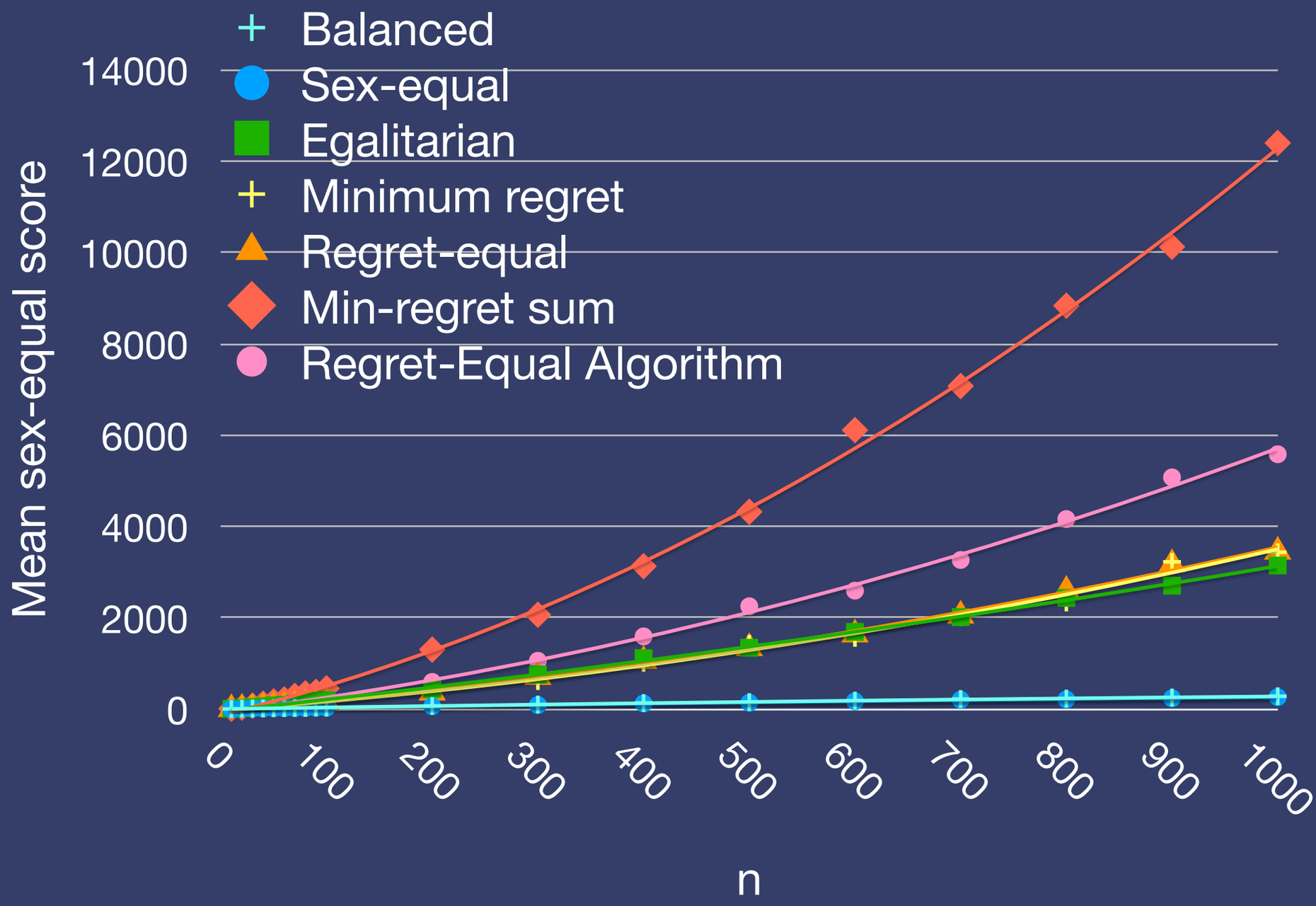


Sex-equal score for different optimal matchings

Sex-equal score for different optimal matchings



Frequency of different optimal stable matchings

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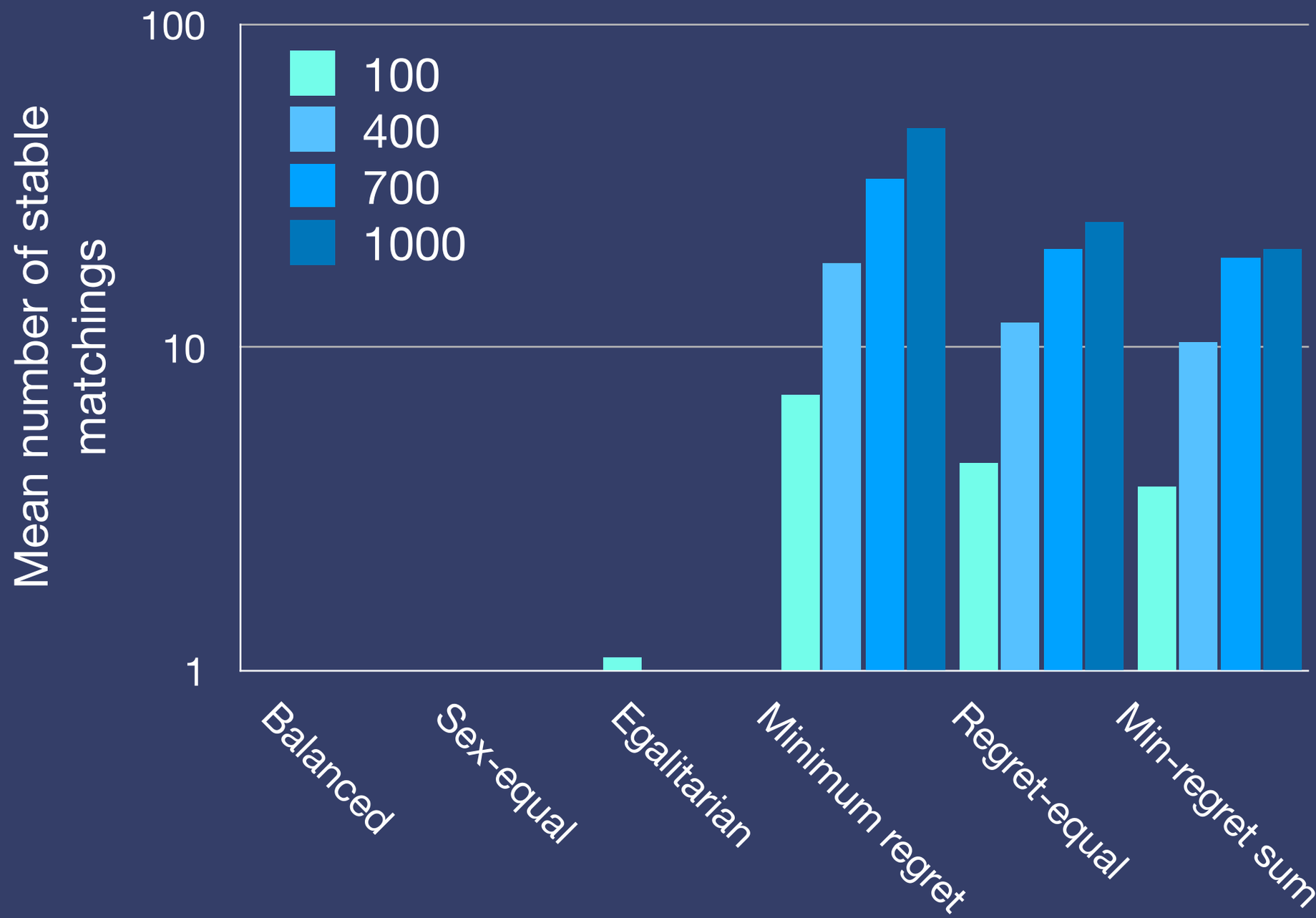

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- Open problem for ‘sex-equality’ -> grouped-women-equality

Thank you

Summary

- Matching problems
- Fairness
- Finding fair stable matchings
- Experiments
- Future work: finding improved algorithms

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