A 3/2-approximation algorithm for the student-project allocation problem with ties

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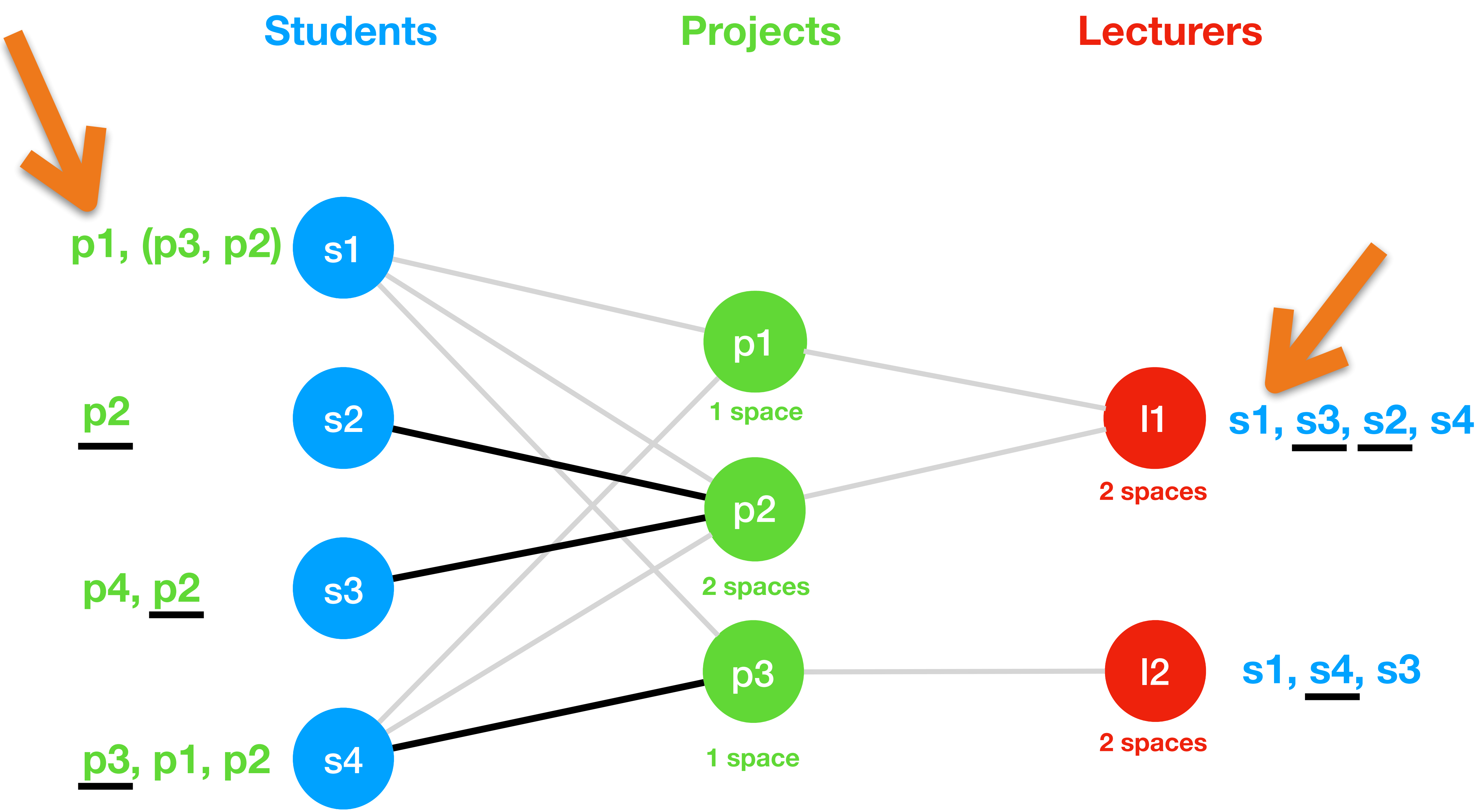
Outline

- Matching problems
- Maximum sized stable matching
  - Integer programming
  - Approximation algorithm
- Future work

Matching Problems

- Assign one set of entities to another set of entities
- Based on preferences and capacities

Student-project allocation problem (SPA-ST)



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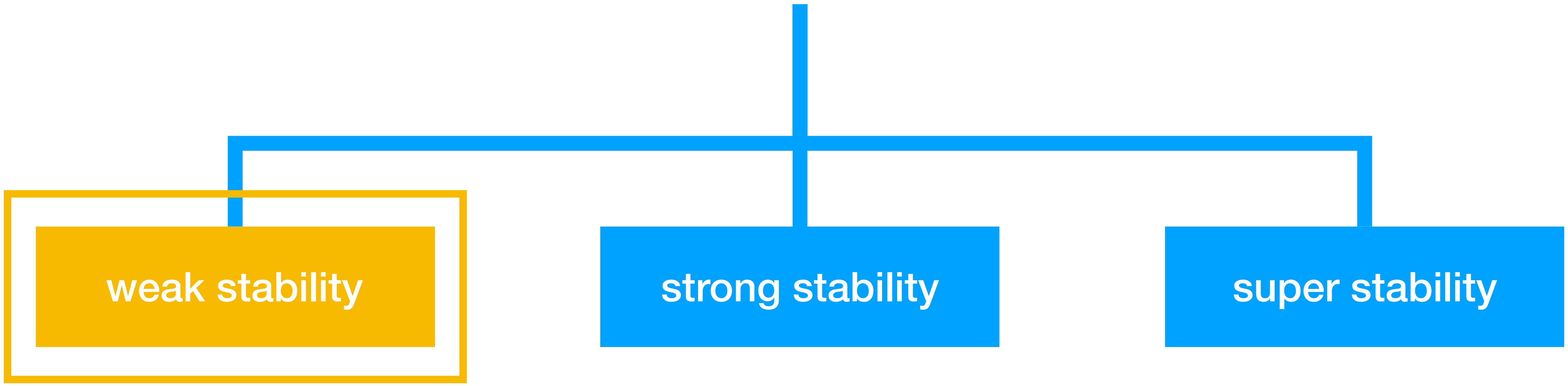
Stable matching



No Blocking pairs: both agents are better off

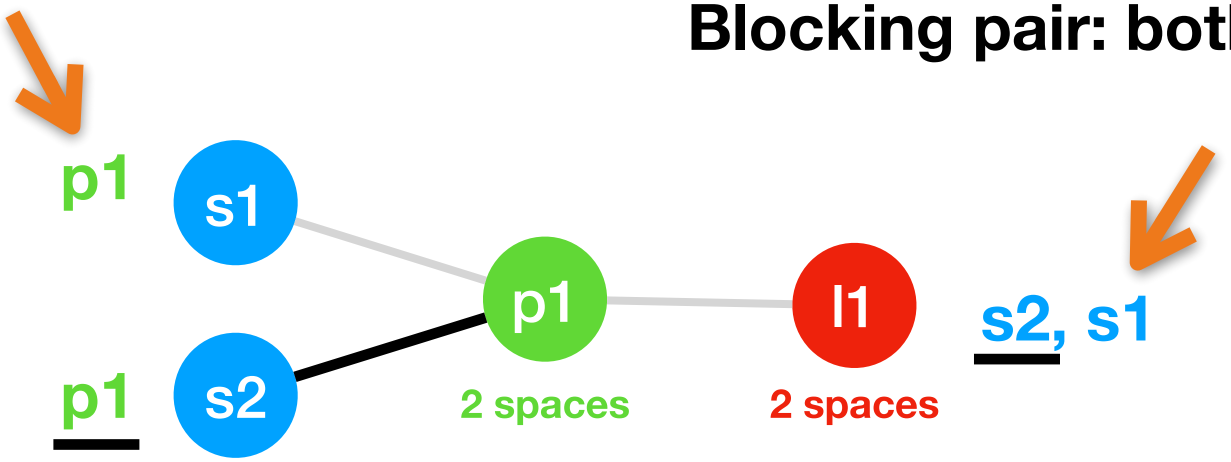
No Blocking Pairs: one agent is better off, the other is no worse off

No Blocking Pairs: neither agent is worse off

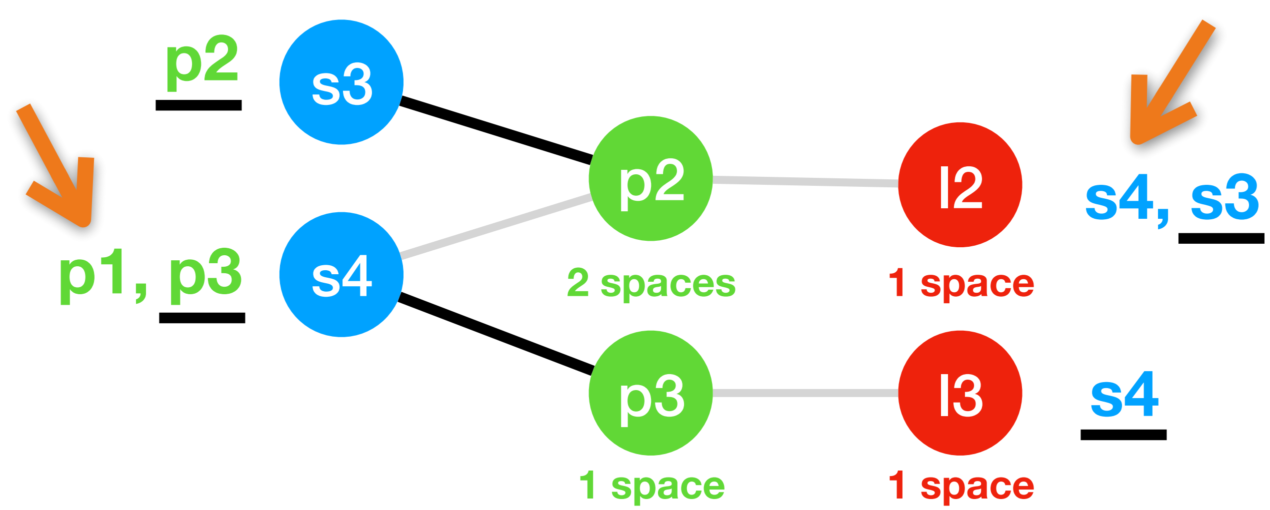
- A **stable matching** is a matching with no blocking pairs

weak stability

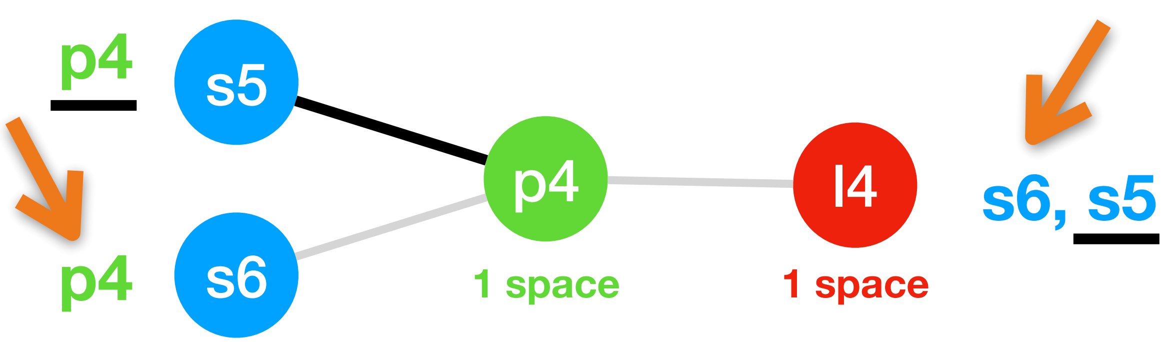
Blocking pair: both agents are better off



project and lecturer undersubscribed



project undersubscribed, lecturer full



project full, (lecturer full or undersubscribed)

Maximum stable matchings

- A **stable matching** is a matching with no blocking pairs
- No ties in preference lists - find a stable matching in polynomial time - all same size
- **Ties in preference lists** - find a stable matching in polynomial time - but stable matchings are **different sizes**
- Finding a maximum sized stable matching is **NP-hard**.

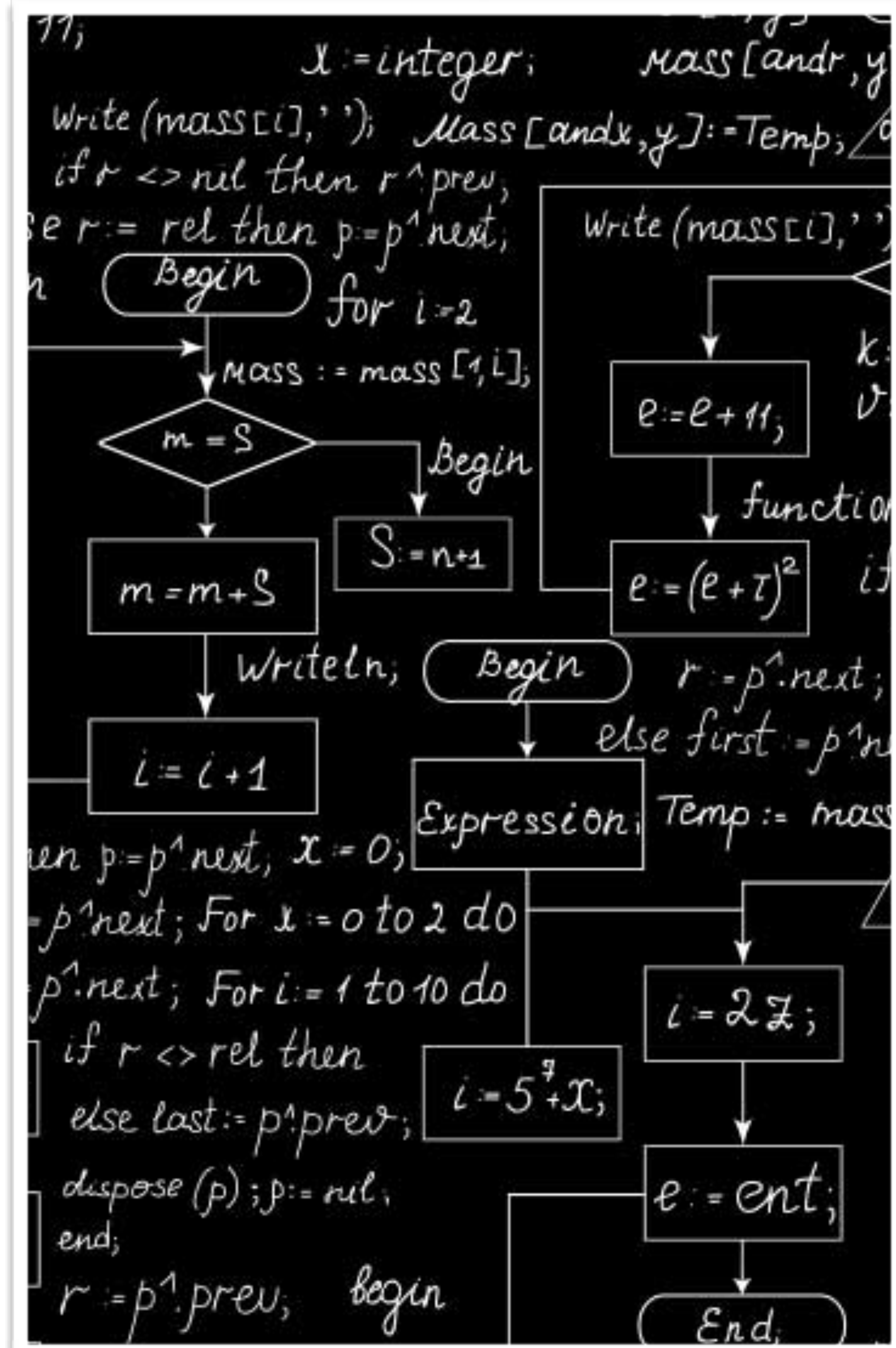
Two Algorithms for the Student Project Allocation Problem; Journal of Discrete Algorithms; 2007; Abraham, Irving, Manlove

Finding a maximum sized stable matching

Two techniques:

1. Approximation algorithm
2. Integer Programming

Approximation Algorithm



Previous work

- Hospitals/Residents with Ties (HRT) - special case of SPA-ST, each lecturer offers one project and the capacity of each lecturer equals the capacity of their offered project
- A 3/2-approximation algorithm exists for HRT
- Can I just convert my problem and use this conversion process?
- Not using a conversion process we tried.

Linear Time Local Approximation Algorithm for Maximum Stable Marriage; Algorithms; 2013; Kiraly

3/2-approximation algorithm

- Created a new 3/2 approximation algorithm for SPA-ST, based on Kiraly's HRT algorithm.
    - Moving from HRT to SPA-ST
        - Lecturers added a lot of complications
        - Definition of a blocking pair is more complicated

Approximation algorithm high-level look

Students (who are not already assigned) apply in turn to their favourite project on their preference list. Assume student **s** applies to project **p**.

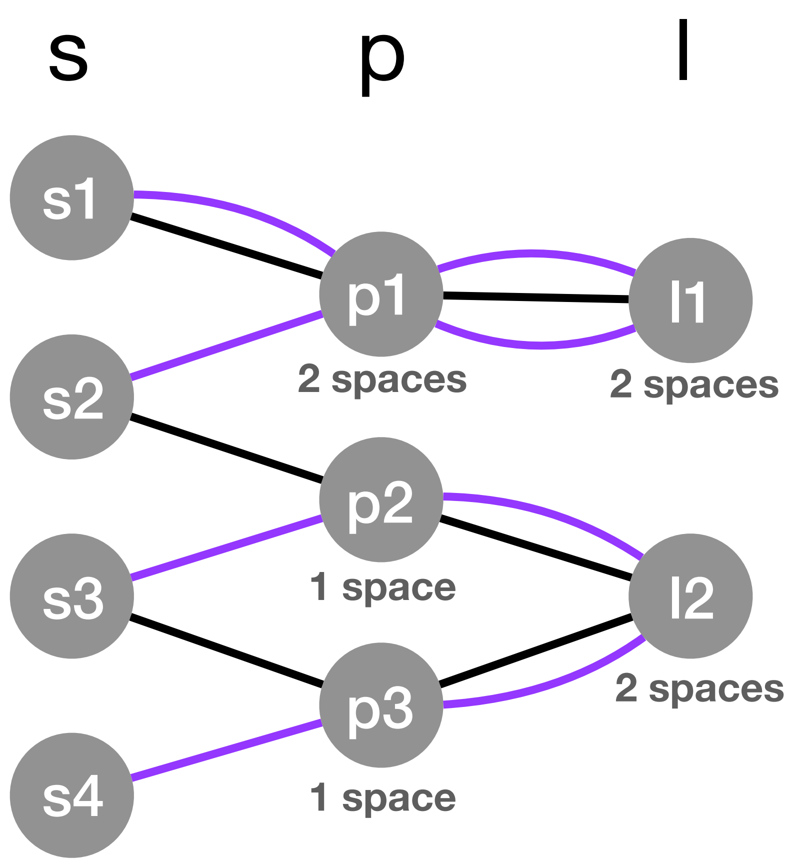
- if **p** and **l** (the lecturer of **p**) are undersubscribed then we add (**s**,**p**) to our matching
- if either **p** or **l** are full then we need to check whether (**s**,**p**) should replace an *existing* pair in the matching
- if there is no chance for **s** to assign to **p** then **s** will remove **p** from their preference list (and will now apply to their next favourite)
- Students iterate twice through their preference list

Proofs

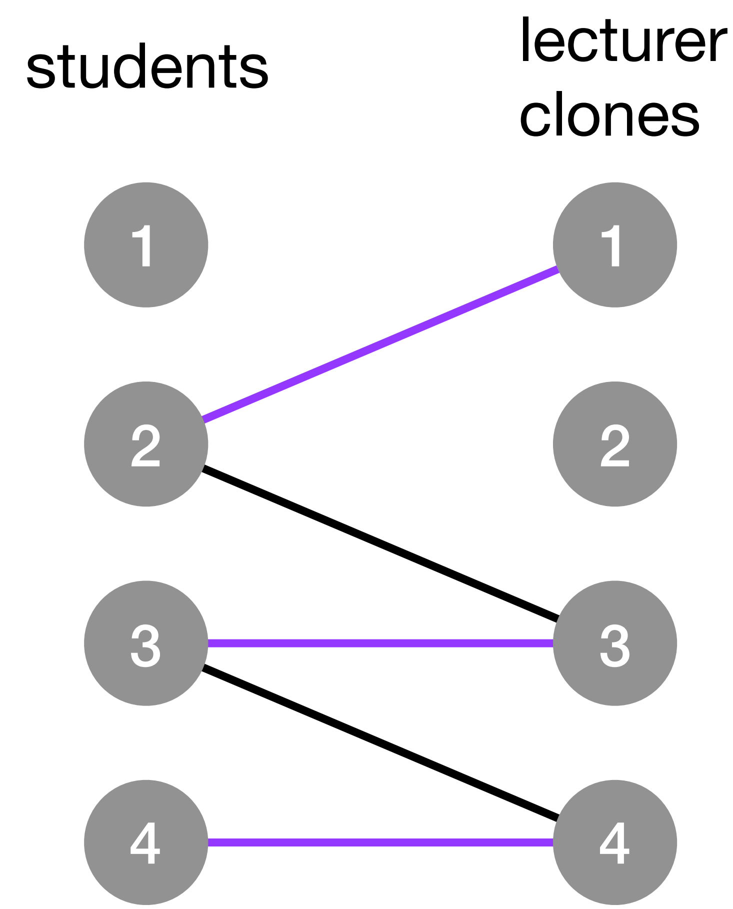
Three proofs required:

- the resultant matching is stable
- the algorithm runs in linear time
- the matching is at least 2/3 the size of optimal

Performance guarantee - creating G'



G



G'

M_{opt}

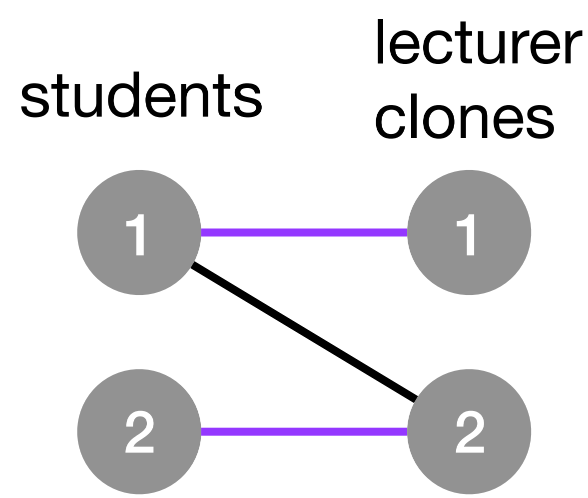
M

M'_{opt}

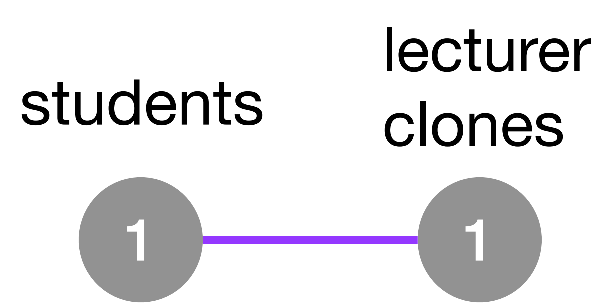
M'

Structures in G'

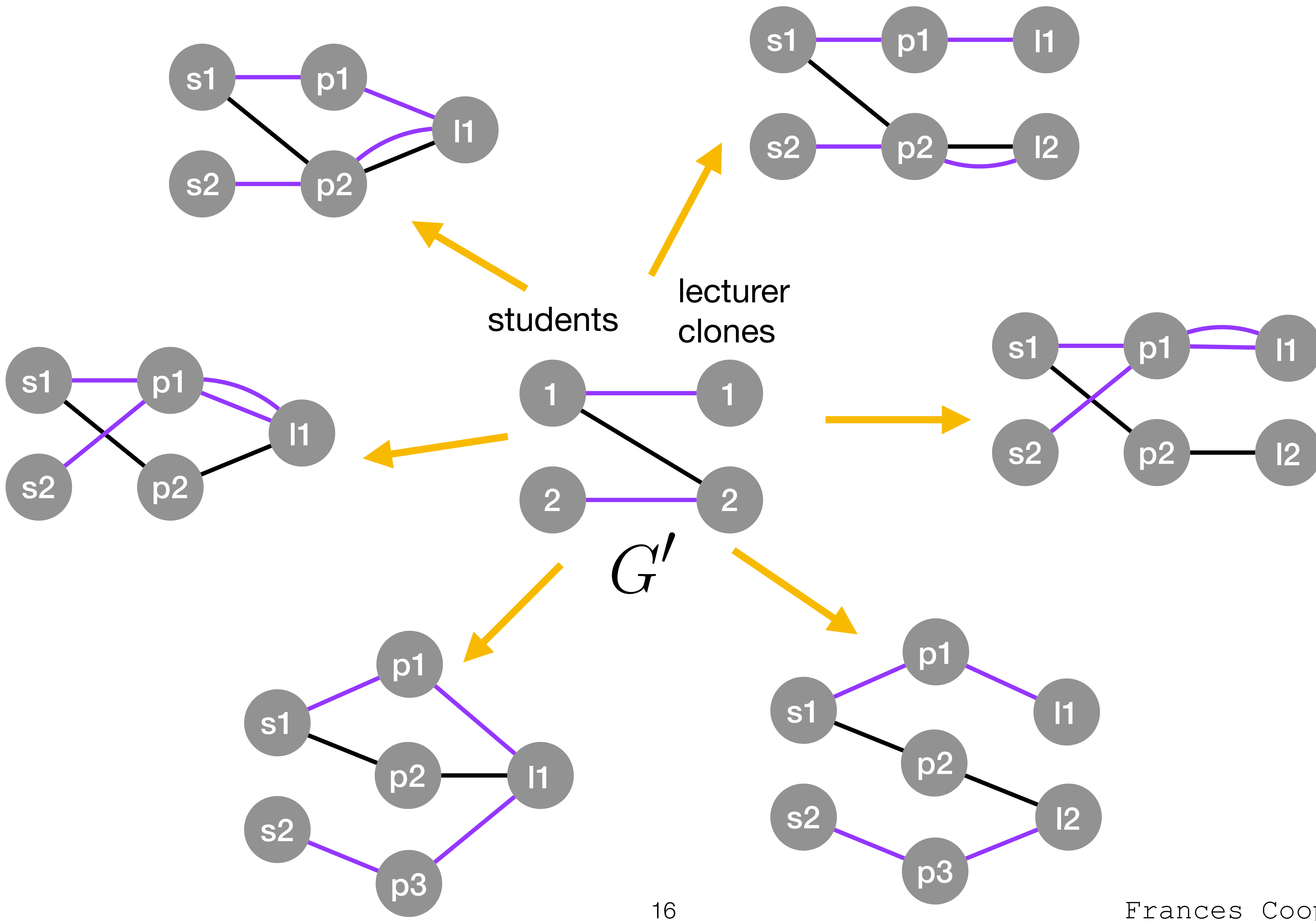
- odd length alternating path with end edges in M'_{opt} (number of edges is 3)



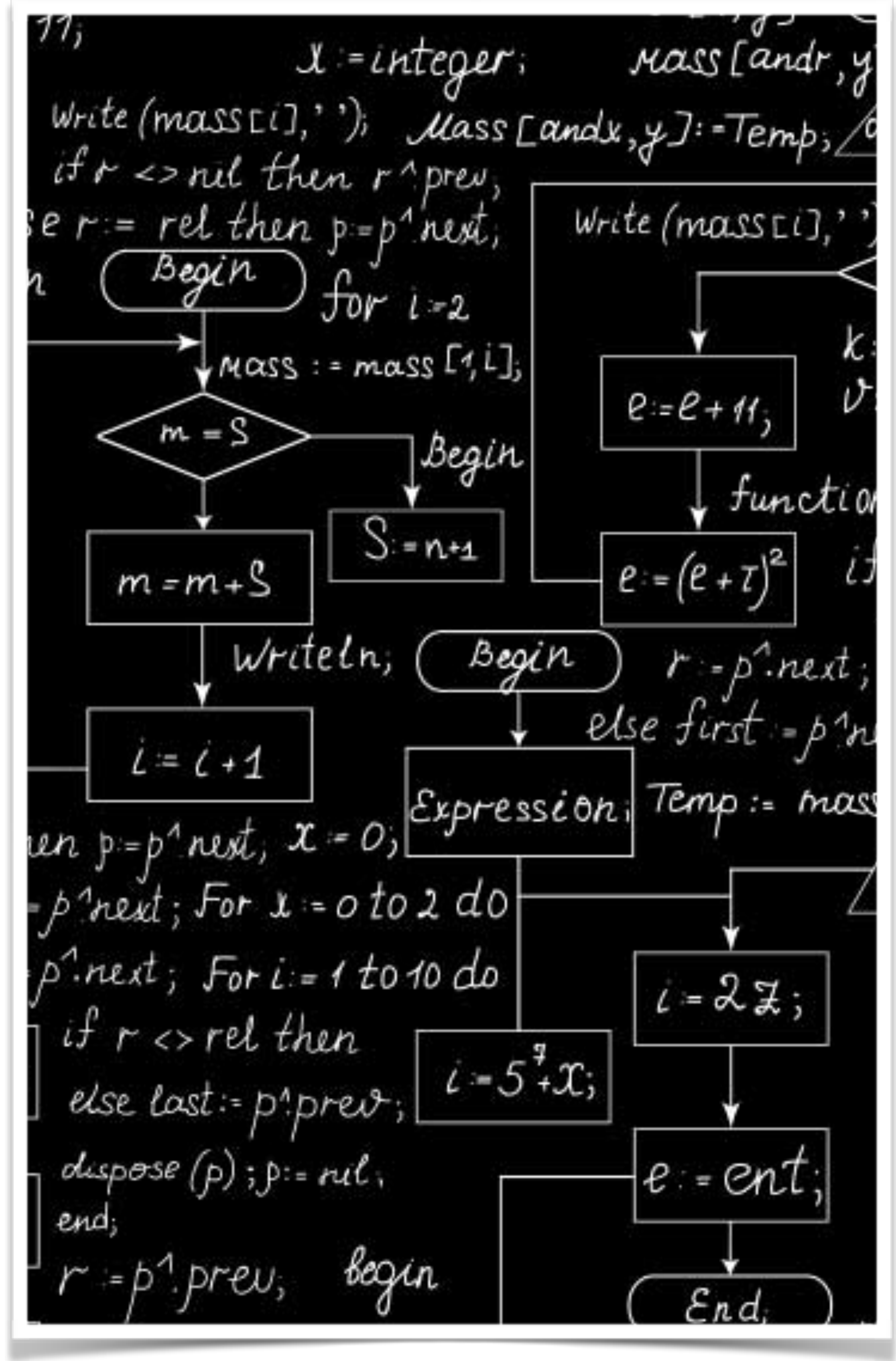
- odd length alternating path with end edges in M'_{opt} (number of edges is 1)



Structures in G'



Integer Program



Integer Programming

- gives an optimal solution
- novel work: stability constraints
- helped in correctness checking
- Gives motivation for using approximation algorithm

Experimental results

Experimental Results

- Java (and Gurobi), 100s of thousands of instances with varying parameters. Ran on approximation algorithm and integer program.
- Does the approximation algorithm stick to 2/3 the size of optimal? Or do we get close to maximum?

Case	minimum A/Max	average size A/Max	Min/Max
TIES1	1.0000	1.000	1.000
TIES2	0.9792	0.997	0.987
TIES3	0.9722	0.993	0.972
TIES4	0.9655	0.990	0.958
TIES5	0.9626	0.986	0.942
TIES6	0.9558	0.984	0.927
TIES7	0.9486	0.982	0.911
TIES8	0.9527	0.980	0.896
TIES9	0.9467	0.980	0.880
TIES10	0.9529	0.982	0.866
TIES11	0.9467	0.984	0.851

- TIES - 10,000 instances per set, 300 students, 250 projects (capacity 420), 120 lecturers (capacity 360), pref lists length 3 to 5.
- increasing prob of student and lecturer ties from 0 to 0.5 in 0.05 steps
- Average approx solution closer to optimal than minimum in all cases

Experimental Results

Scalability

- SCALS - 10,000 students up to 50,000 students. Pref lists 3 to 5 and ties 0.2
- SCALP - 500 students, ties 0.4, Pref lists increased from 25 to 150 in steps of 25.
- much faster than using the integer program

	instances completed		average total time (ms)	
Case	A	Max	A	Max
SCALS1	10	10	1393.8	227764.3
SCALS2	10	9	5356.7	1096045.6
SCALS3	10	0	13095.3	N/A
SCALS4	10	0	18883.5	N/A
SCALS5	10	0	20993.0	N/A
SCALP1	10	9	193.3	94242.9
SCALP2	10	10	189.4	631225.2
SCALP3	10	3	196.6	882251.0
SCALP4	10	1	248.5	1594201.0
SCALP5	10	0	283.7	N/A
SCALP6	10	0	288.4	N/A

Experimental Results

- So is it worth using?

coram better chances for children since 1739

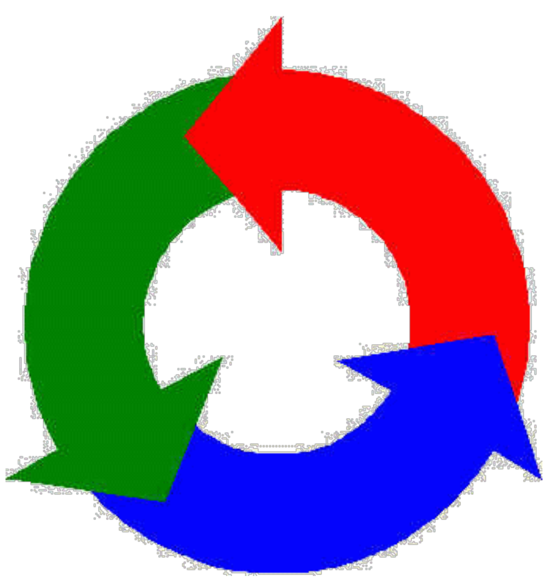
- Coram - assigning adopted children to families. ~ 100's of agents. Preference lists long and probability of ties high

- 21 instances, increasing difficulty. IP could only solve first 6 within 5 minutes, approximation algorithm took less than 2 seconds for each

Future Work

- Finding an approximation algorithm with a better performance guarantee than 3/2
- Finding a better inapproximability result than 33/29
- coalitions:
    - group of several students and lecturers
    - permute their assignments
    - some or all get a better outcome

Approximation Algorithms for Stable Matching Problems; PhD thesis; 2007; H. Yanagisawa



Thank you

Summary

- Student-project allocation problem
- Finding a maximum stable matching
  - Integer programming
  - Approximation algorithm
- Future work: improved performance guarantee; improved inapproximability result; coalitions

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